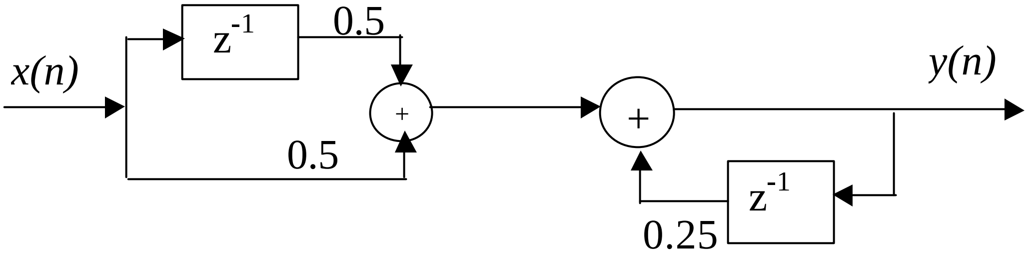
**January 18, 2002**
**Class ECE308-03: 3.3**

*Example:*

Sketch the block diagram representation of the discrete-time system described by the input-output relation

$$y(n) = \frac{1}{4}y(n-1) + \frac{1}{2}x(n) + \frac{1}{2}x(n-1)$$



## Classification of Discrete-Time Systems

## Static System

If the output depends at most on the input sample at the same time, but not on past or future samples of the input, is called static or memoryless system.

$$y(n) = ax(n)$$
$$y(n) = nx(n) + bx^3(n)$$

They don’t need to store any of the past input or output in order to compute the present output. It can be described in the following form

$$y(n) = t\left[x(n), n\right]$$

**Dynamic System**

If the output depends on the any past or future samples of the input, the system is called dynamic or system with memory.

$$y(n) = ax(n) + 3x(n-1)$$
$$y(n) = \sum_{k=0}^{n} x(n-k)$$
$$y(n) = \sum_{k=0}^{\infty} x(n-k)$$

**Time-invariant Systems:**

A system is called time-invariant if its input-output characteristics do not change with time.

$$x(n) \overset{t}{\rightarrow} y(n)$$
$$x(n-k) \overset{t}{\rightarrow} y(n-k)$$

It is also called shift invariant.

We can write the output

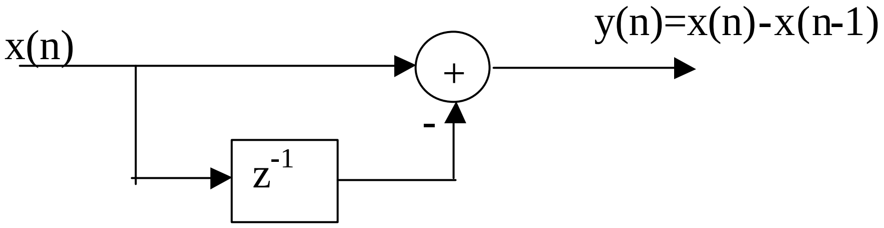
$$y(n,k) = t\left[x(n-k)\right]$$

So, if the output $y(n,k) = y(n-k)$, for all possible values of k, the system is time invariant.

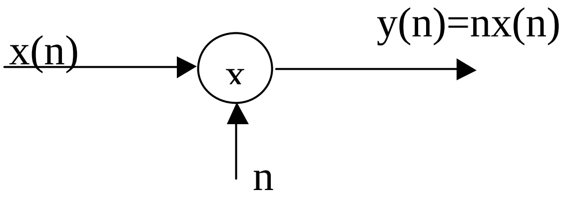
**Time-variant systems:**

If the output $y(n,k) \neq y(n-k)$, even for one value of k, the system is called time variant.

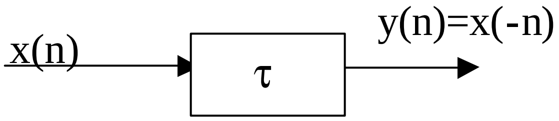
Examples of time-invariant and time-variant systems



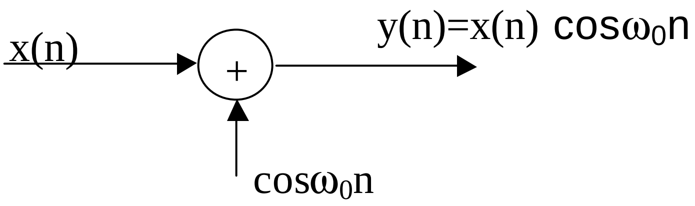
Differentiator



Time multiplier



Folder



Modulator

*1. Differentiator systems* are a time-invariant, because;

The system equation:

$$y(n) = t\left[x(n)\right] = x(n) - x(n-1)$$

If the input delay by k unit

$$y(n,k) = x(n-k) - x(n-k-1)$$

The other side if we delay y(n) by k unit

$$y(n-k) = x(n-k) - x(n-k-1)$$

So, two equations are equal, therefore the system is time invariant.

2. *Time multiplier systems* are time variant, because;

The system equation is

$$y(n) = t\left[x(n)\right] = nx(n)$$

If the input delay k unit

$$y(n,k) = t\left[x(n-k)\right] = nx(n-k)$$

If we delay y(n) by k unit in time,

$$y(n-k) = (n-k)x(n-k)$$

$$= nx(n-k) - kx(n-k)$$

The system is time variant, since $y(n,k) \neq y(n-k)$

3. *Folder systems* are time variant, because:

The system equation is

$$y(n) = t\left[x(n)\right] = x(-n)$$

If the input delay k unit

$$y(n,k) = \left[x(n-k)\right] = x(-n-k)$$

If we delay y(n) by k unit in time,

$$y(n-k) = x(-n+k)$$

The system is time variant, since $y(n,k) \neq y(n-k)$.

4. *Modulator systems* are time variant, because:

The system equation is

$$y(n) = x(n)\cos w_0 n$$

If the input delay k unit

$$y(n,k) = x(n-k)\cos w_0 n$$

If we delay y(n) by k unit in time,

$$y(n-k) = x(n-k)\cos w_0 (n-k)$$

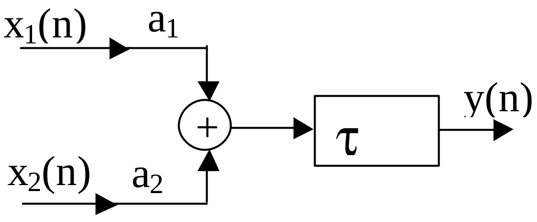
The system is time variant, since $y(n,k) \neq y(n-k)$.

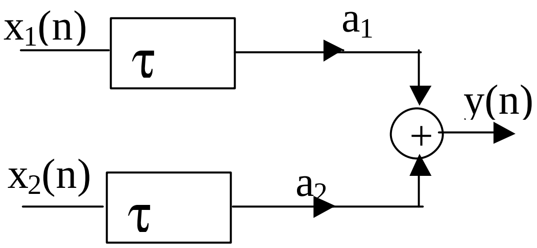
**Linear System:**

A system is linear if and only if

$$t\left[a_1 x_1(n) + a_2 x_2(n)\right] = a_1 t\left[x_1(n)\right] + a_2 t\left[x_2(n)\right]$$
$$= a_1 y_1(n) + a_2 y_2(n)$$

for any arbitrary $x_1(n)$ and $x_2(n)$, and any arbitrary constant $a_1$ and $a_1$. It satisfies the superposition principle.





This relation demonstrates the additivity property of a linear system.

In general, we can write that

$$x(n) = \sum_{k=1}^{M-1} a_k x_k(n) \quad \xrightarrow{t} \quad y(n) = \sum_{k-1}^{M-1} a_k y_k(n)$$

where $y_k(n) = t\left[x_k(n)\right],\ k = 1, 2, \ldots, M-1$

**Causal Systems:**
A system is called *causal* if the output of the system at any time n depends only on present and past inputs, but not depends on future inputs.

$$y(n) = F[x(n), x(n-1), x(n-2), ...]$$

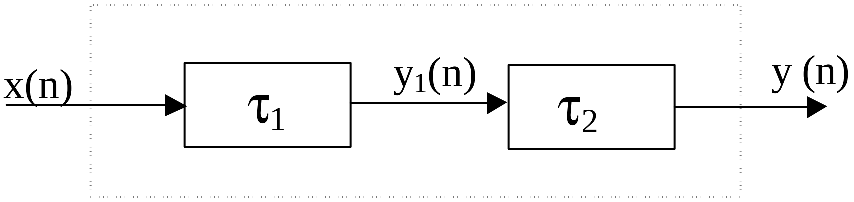
If the system does not satisfy this definition, it is called *noncausal*.

Stable Systems:
An arbitrary relaxed system is said to be bounded input-bounded output (BIBO) stable if and only if every bounded input produces abounded output.

$$|x(n)| \leq M_x < \infty \qquad |y(n)| \leq M_y < \infty$$

for all n. $M_x$ and $M_x$ are some finite numbers.

**Interconnection of Discrete-Time Systems**
Discrete-time systems can be interconnected to form larger systems. They can be interconnected serial or parallel.



$$\begin{aligned} y_1(n) &= t_1[x(n)] \\ y(n) &= t_2[y_1(n)] \\ &= t_2\left[t_2[x(n)]\right] \end{aligned}$$

If we combine $\tau_1$ and $\tau_2$ to $t_c = t_2 t_1$, then

$$y(n) = \tau_c[x(n)]$$

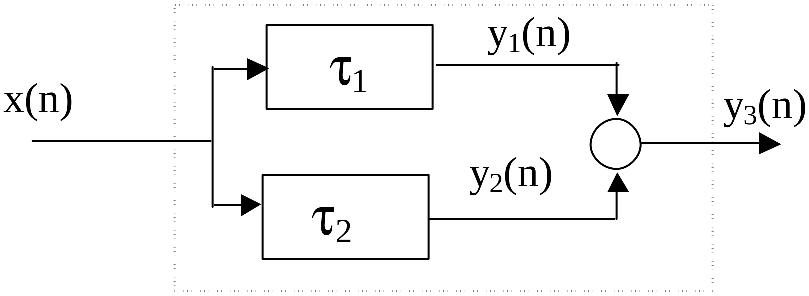
If the systems $\tau_1$ and $\tau_2$ are linear and time invariant

$$\tau_2\tau_1 = \tau_1\tau_2$$

otherwise

$$\tau_2\tau_1 \neq \tau_1\tau_2$$

In the parallel interconnection systems,



$$\begin{aligned} y_3(n) &= y_1(n) + y_2(n) \\ &= \tau_1[x(n)] + \tau_2[x(n)] \\ &= (\tau_1 + \tau_2)[x(n)] \\ &= \tau_p[x(n)] \end{aligned}$$

where $\tau_p = \tau_1 + \tau_2$