January 16, 2002
Class ECE308-03: 3.2

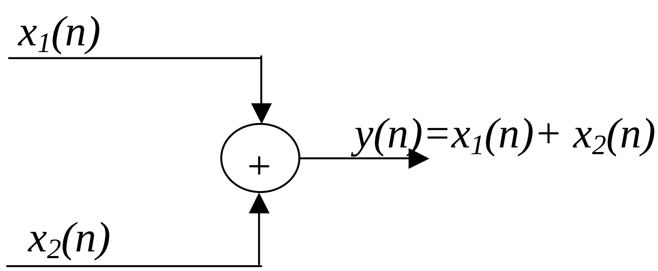
Addition, Multiplication, and scaling of sequences

Amplitude Scaling: (A Constant Multiplier)

$$y(n) = Ax(n), \quad -\infty < n < \infty$$

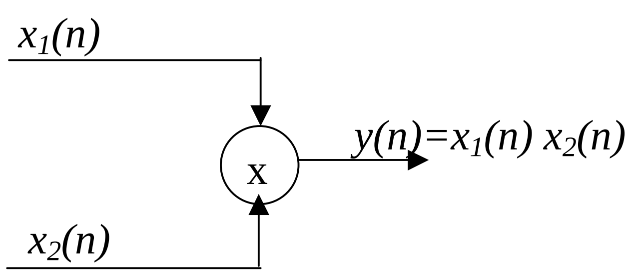
Addition of two signals (An Adder)

$$y(n) = x_1(n) + x_2(n), \quad -\infty < n < \infty$$

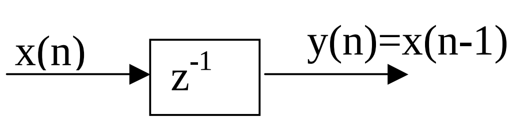


The product of two signals (A signal Multiplier)

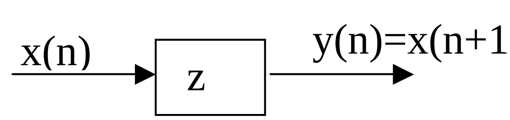
$$y(n) = x_1(n)x_2(n), \quad -\infty < n < \infty$$



A unit delay element

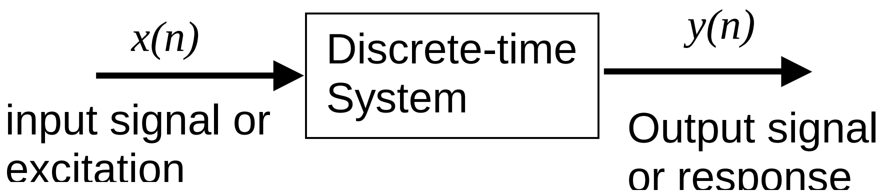


A unit advance element



2.2.1 Input-Output Description of Systems

The relation between the input and output signals are known input-output relationship



Mathematical representation of the transformation is

$$y(n) = \tau\left[x(n)\right]$$

τ denotes the transformation. In general input-output relationship can be also shown as

$$x(n) \overset{\tau}{\rightarrow} y(n)$$

Example:

The input signal is

$$x(n) = \begin{cases} |n| & -3 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

a. $y(n) = x(n)$

b. $y(n) = x(n-1)$

c. $y(n) = x(n+1)$

d. $y(n) = \frac{1}{3}\left[x(n+1) + x(n) + x(n-1)\right]$

e. $y(n) = \max\left[x(n+1), x(n), x(n-1)\right]$

f. $y(n) = \sum_{k=-\infty}^{n} x(k) = x(n) + x(n-1) + x(n-2) + \ldots$

Solution:

$$x(n) = \{\ldots, 0, 3, 2, 1, \underset{\uparrow}{0}, 1, 2, 3, 0, \ldots\}$$

a. $y(n) = x(n)$, $y(n) = \{\ldots, 0, 3, 2, 1, \underset{\uparrow}{0}, 1, 2, 3, 0, \ldots\}$

b. $y(n) = x(n-1)$, $y(n) = \{\ldots, 0, 3, 2, \underset{\uparrow}{1}, 0, 1, 2, 3, 0, \ldots\}$

c. $y(n) = x(n+1)$, $y(n) = \{\ldots, 0, 3, 2, 1, 0, \underset{\uparrow}{1}, 2, 3, 0, \ldots\}$

d. $y(n) = \frac{1}{3}\left[x(n+1) + x(n) + x(n-1)\right]$

$$y(0) = \frac{1}{3}\left[x(1) + x(0) + x(-1)\right] = \frac{1}{3}\left[1 + 0 + 1\right] = \frac{2}{3}$$

$$y(n) = \left\{\ldots, 0, 1, \frac{5}{3}, 2, 1, \underset{\uparrow}{\frac{2}{3}}, 1, 2, \frac{5}{3}, 1, 0, \ldots\right\}$$

e. $y(n) = \max\left[x(n+1), x(n), x(n-1)\right]$

$$y(0) = \max\left[x(1), x(0), x(-1)\right] = \max\left[1, 0, 1\right] = 1$$

$$y(n) = \{0, 3, 3, 3, 2, \underset{\uparrow}{1}, 2, 3, 3, 3, 0, \ldots\}$$

f. $y(n) = \sum_{k=-\infty}^{n} x(k) = x(n) + x(n-1) + x(n-2) + \ldots$

This system is called an accumulator.

$$y(0) = \sum_{k=-\infty}^{0} x(k) = x(0) + x(-1) + x(-2) + x(-3)$$

$$= 0 + 1 + 2 + 3 = 6$$

$$y(n) = \{\ldots, 0, 3, 5, 6, \underset{\uparrow}{6}, 7, 9, 12, 12, \ldots\}$$

A simple algebraic manipulation the input-output relation of the accumulator can be written as

$$y(n) = \sum_{k=-\infty}^{n} x(k) = \sum_{k=-\infty}^{n-1} x(k) + x(n)$$

$$y(n) = y(n-1) + x(n)$$

which justifies the term *accumulator*.
The current value of the output is found by adding the current value of the input to the previous output value.

Example:

$x(n) = nu(n)$

Find the output under the following condition

a. $y(-1) = 0$

b. $y(-1) = 1$

Solution:
The output of the system is defined as

a. $$y(n) = \sum_{k=-\infty}^{n} x(k) = \sum_{k=-\infty}^{-1} x(k) + \sum_{k=0}^{n} x(k)$$

$$y(n) = y(-1) + \sum_{k=0}^{n} x(k)$$

$$y(n) = 0 + \sum_{k=0}^{n} k = \frac{n(n+1)}{2} \quad \text{, for } n \geq 0$$

b. $$y(n) = y(-1) + \sum_{k=0}^{n} x(k)$$

$$y(n) = 1 + \sum_{k=0}^{n} k = 1 + \frac{n(n+1)}{2} = \frac{n^2 + n + 2}{2}, \quad \text{for } n \geq 0$$