January 14, 2002
Class # 3.1

Chapter 2

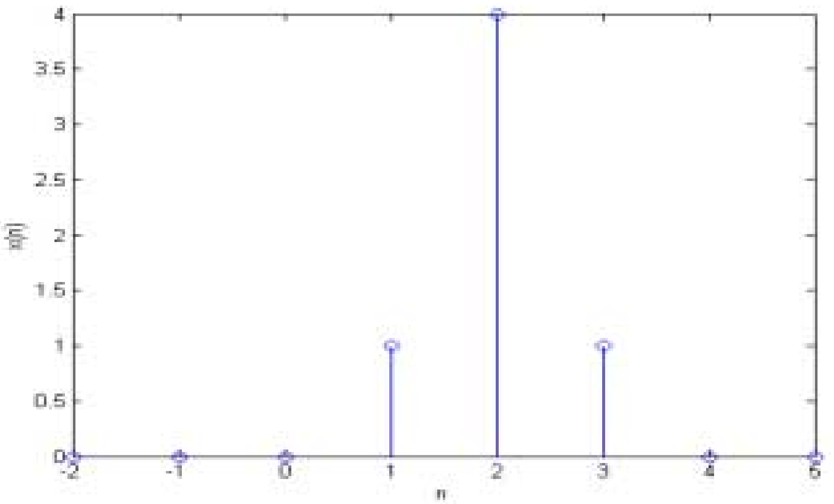
Discrete-Time Signals and Systems

2.1 Discrete-Time Signals

A Discrete-time signal $x(n)$ is a function of an independent integer variable n. The signal $x(n)$ is not defined for noninteger values of n.

We can represent a discrete time signal differently;

1. Graphical representation



2. Functional representation, such as

$$x(n) = \begin{cases} 1, & \text{for } n = 1,3 \\ 4, & \text{for } n = 2 \\ 0, & \text{elsewhere} \end{cases}$$

3. Tabular representation, such as

n	…	-2	-1	0	1	2	3	4	5	…
X(n)	…	0	0	0	1	4	1	0	0	…

4. Sequential representation such as

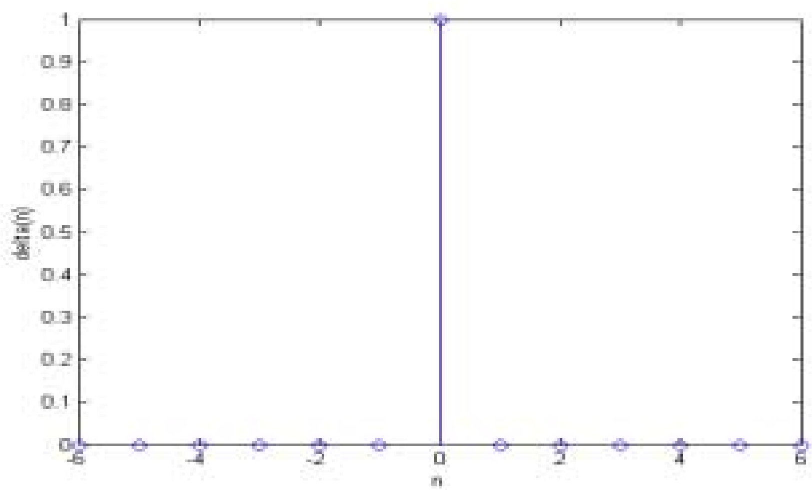
$$x(n) = \{\ldots 0, \underset{\uparrow}{0}, 1, 4, 1, 1, 0, 0, \ldots\}$$

The time origin (n=0) is indicated by the symbol ↑.

2.2.1 Some Elementary Discrete-Time Signals

1. The unit sample sequence:

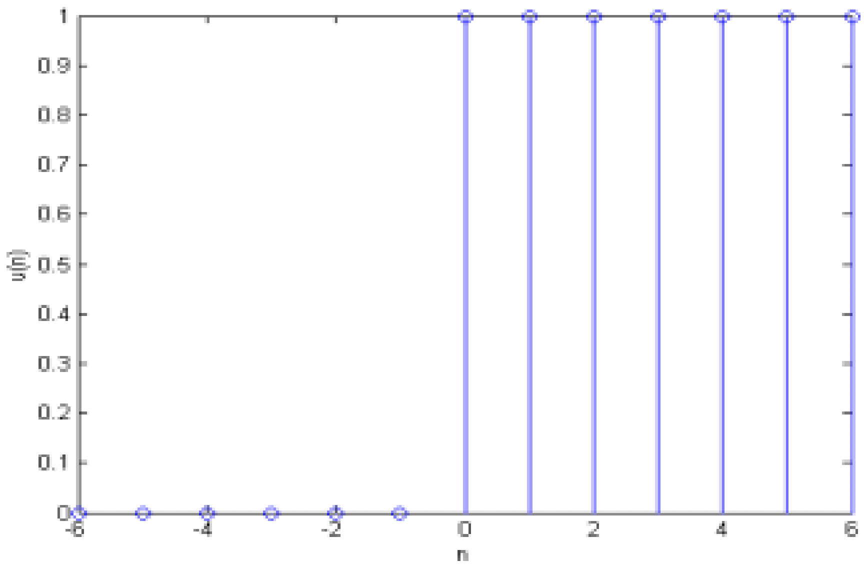
$$\delta(n) = \begin{cases} 1, & \text{for } n = 0, \\ 0, & \text{for } n \neq 0 \end{cases}$$



The unit sample sequence is often referred to as a discrete-time impulse or an impulse.

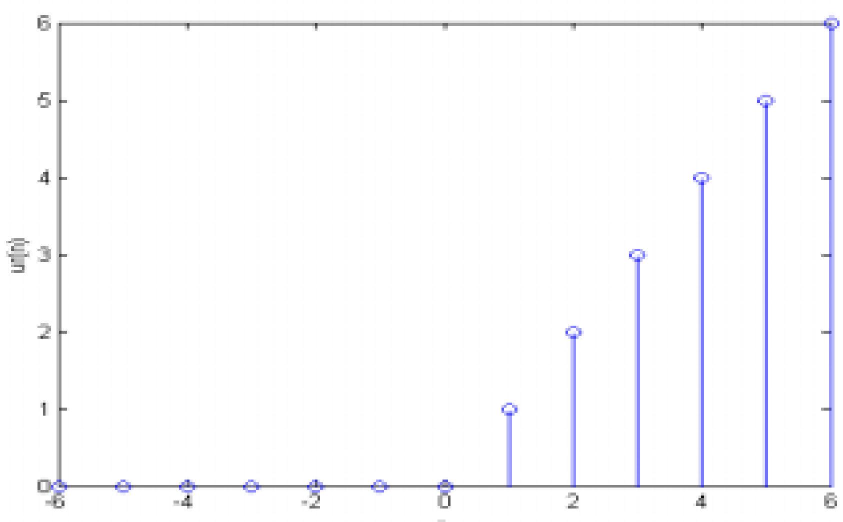
2. The unit step signal

$$u(n) = \begin{cases} 1, & \text{for } n \geq 0, \\ 0, & \text{for } n < 0 \end{cases}$$



3. The unit ramp signal

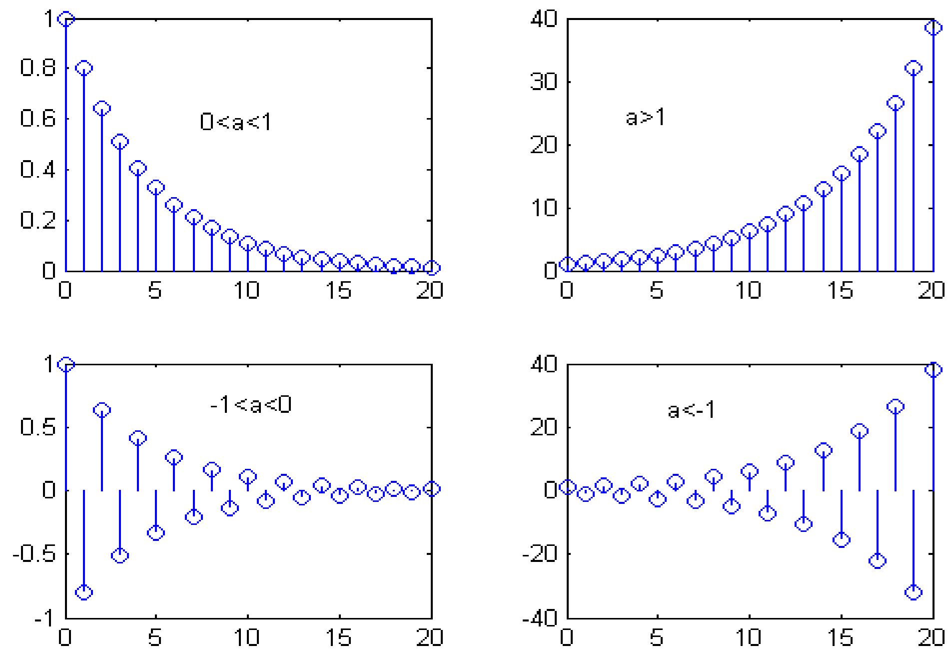
$$u_r(n) = \begin{cases} n, & \text{for } n \geq 0, \\ 0, & \text{for } n < 0 \end{cases}$$



4. The exponential signal

$$x(n) = a \quad \text{for all } n$$

If a is real, then $x(n)$ is a real signal.



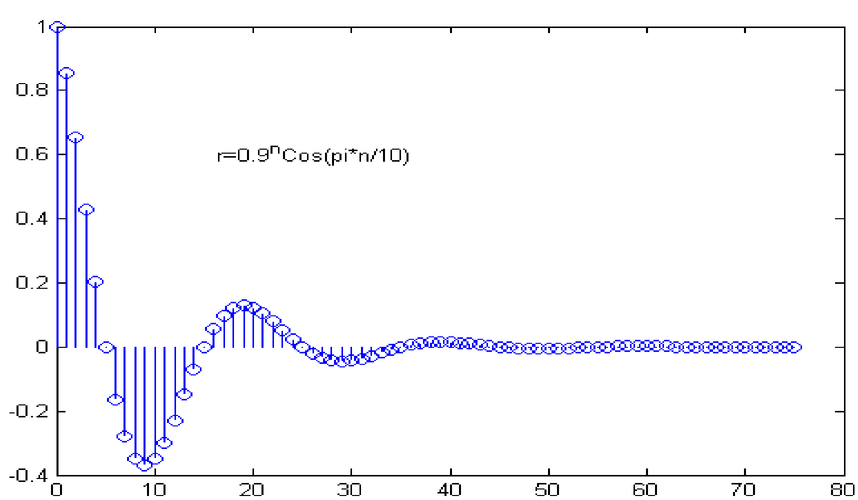
If a is complex, $x(n)$ can be expressed as

$$x(n) = a^n = \left(re^{j\theta}\right) = r^n e^{j\theta n}$$

$$= r^n(\cos\theta n + j\sin\theta n)$$

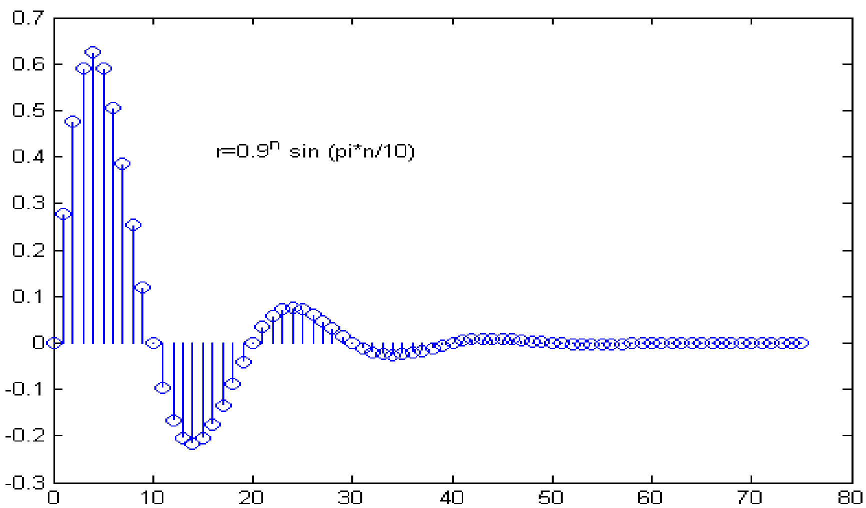
The real part is

$$x_R(n) = r^n \cos\theta n$$



The imaginary part

$$x_R(n) = r^n \sin\theta n$$

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>> n=0:99
>>
x=(0.9.^n).*sin(pi.*n/10);
>> stem(n,x)
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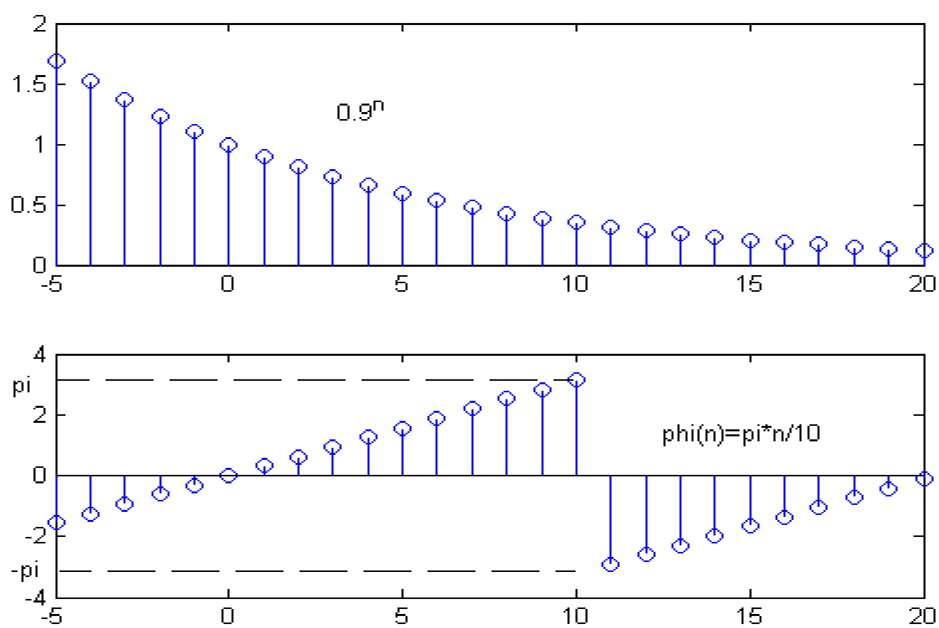
Or we can represent the amplitude function

$$|x(n)| = r^n$$

and the phase function

$$\angle x(n) = \phi(n) = \theta n$$



Energy signals and Power signals

The energy E of a signal $x(n)$ is defined as

$$E = \sum_{n=-\infty}^{\infty} \left| x(n) \right|^2$$

If E is finite $(0 < E < \infty)$ then $x(n)$ is called an energy signal.

The average power of $x(n)$ is defined as

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left| x(n) \right|^2$$

The energy of $x(n)$ is be found over the finite interval $-N < n < N$ as

$$E_N = \sum_{n=-N}^{N} \left| x(n) \right|^2$$

The signal energy E is

$$E = \lim_{N \to \infty} E_N$$

The average power of $x(n)$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} E_N$$

Periodic Signals

A signal $x(n)$ is periodic with period N(N>0) if and only if

$x(n+N) = x(n)$ for all n.

The sinusoidal signal

$$x(n) = A \sin 2\pi f_0 n$$

is periodic when f_0 is a rational number, that can be expressed as

$$f_0 = \frac{k}{N}$$

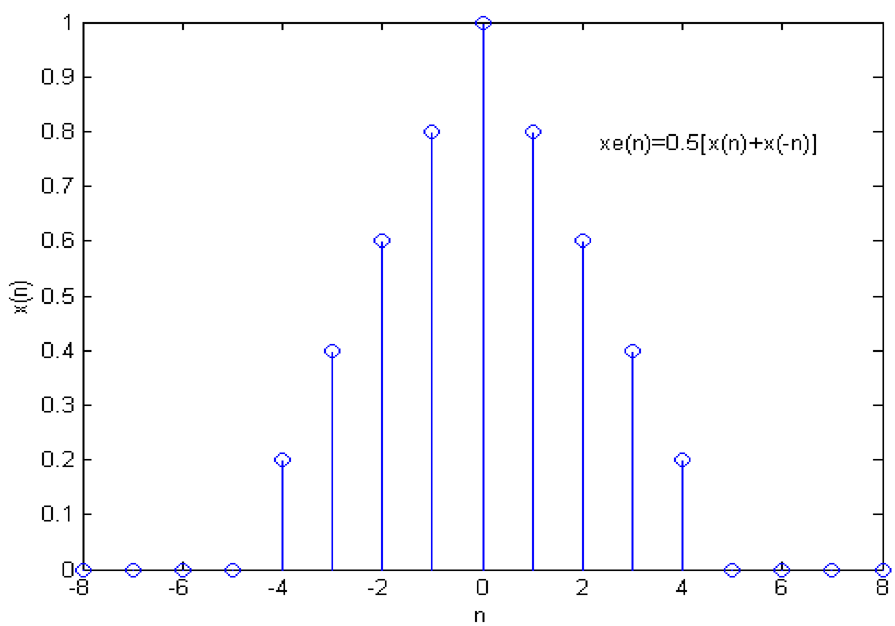
where k and N are integer.
The average power in periodic $x(n)$ is given by

$$P = \frac{1}{N}\sum_{n=0}^{N-1}|x(n)|^2$$

Symmetric (even) and antisymmetric (odd) signals

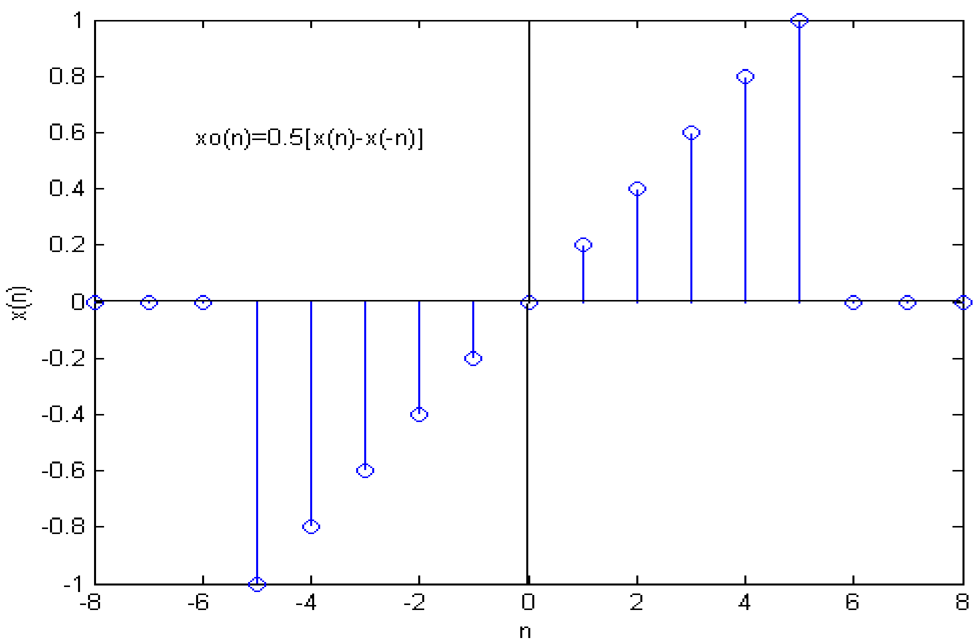
A real-valued signal $x(n)$ is called symmetric (even) if

$$x(-n) = x(n)$$



A signal $x(n)$ is called antisymmetric (odd) if

$$x(-n) = -x(n)$$



2.1.3 Simple Manipulation of $x(n)$

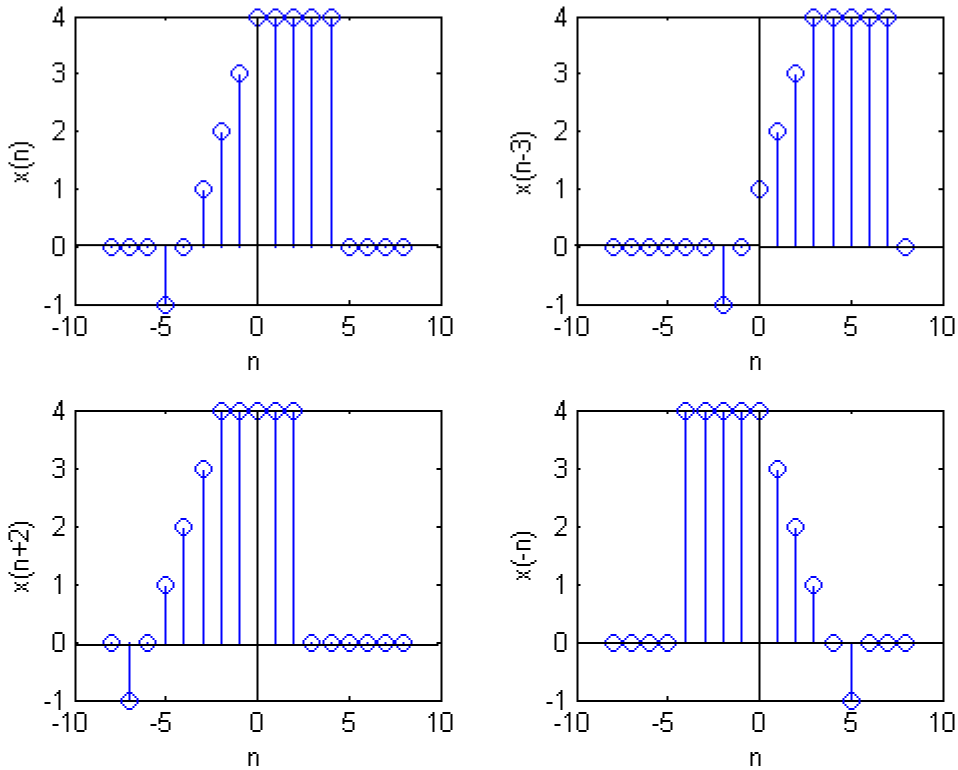
Transformation of the time

A signal $x(n)$ may be shifted in time replacing the time variable n by n-k, where k is integer.

If k is a positive integer, the results in a delay of the signal by k unit of time

If k is a negative integer, the results in a advance of the signal by k unit of time

If the time base is to replace the independent variable n by –n, It is called folding or a reflection of the signal the time origin n=0.

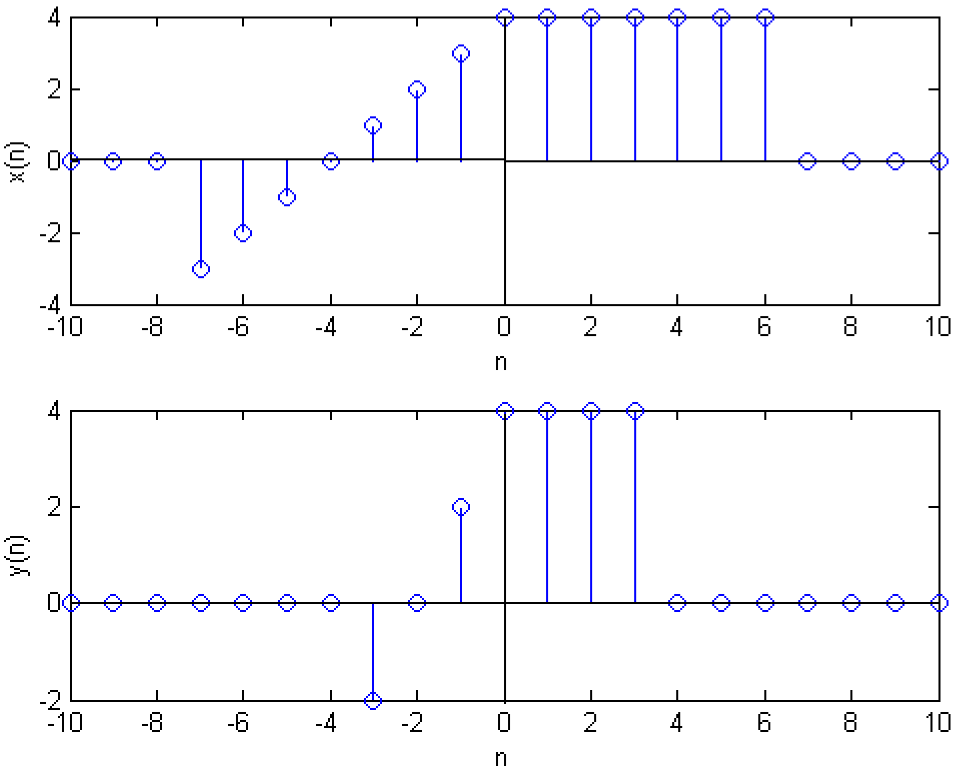


Let's denote the Time-delay operation by TD and the folding operation by FD

$$TD_k\left[x(n)\right]=x(n-k),\quad k>0$$
$$FD\left[x(n)\right]=x(-n)$$
$$TD_k\left\{FD[x(n)]\right\}=TD_k\left[x(-n)\right]=x(-n+k)$$
$$FD\left\{TD_k[x(n)]\right\}=FD\left[x(n-k)\right]=x(-n-k)$$

Example:

x(n)=[0,0,0,-3,-2,-1,0,1,2,3,4,4,4,4,4,4,4,0,0,0]
$y(n)=x(2n)$
Find y(n).
$y(0)=x(0),\ y(-1)=x(-2),\ y(1)=x(2),\ y(-2)=x(-4),\ y(2)=x(4)$


x(n)
n
y(n)
n