Quantization of Continuous-Amplitude Signals.

Converting a discrete-time continuous-amplitude signal into a digital signal by expressing each sample value as a finite number of digits, is called *quantization*.
The error between continuous-valued signal and a finite set of discrete value levels signal is called *quantization error* or *quantization noise*.

The output of quantizer is

$$x_q(n) = Q[x(n)]$$

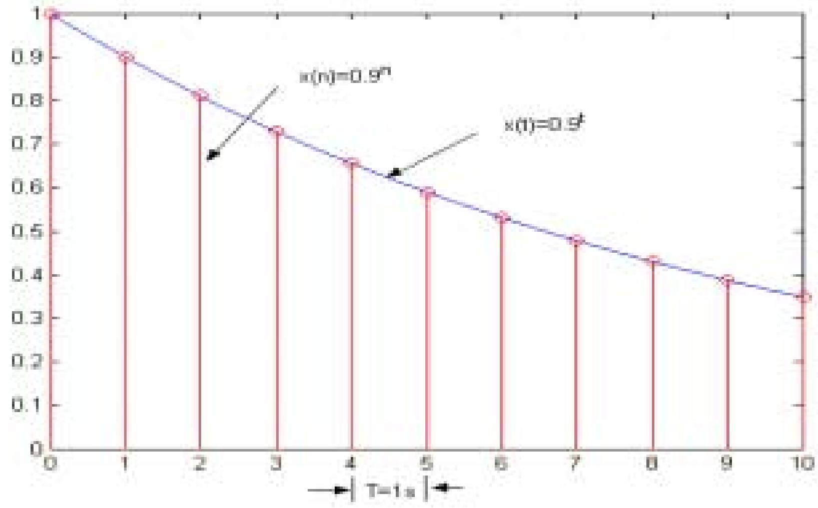
The quantizer error is

$$e_q(n) = x_q(n) - x(n)$$

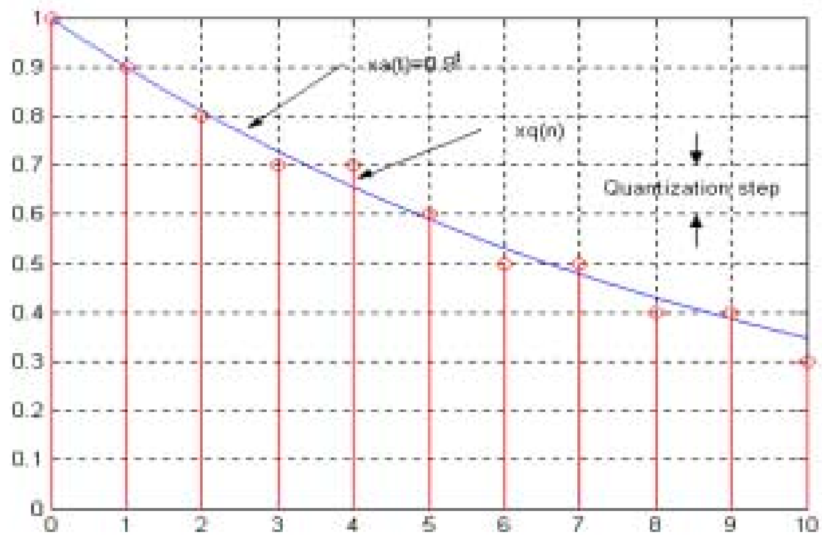
Example:
Let's consider the discrete-time signal as

$$x(n) = \begin{cases} 0.9^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

The sampling frequency is $F_s = 1Hz$.




```
>> t=0:0.01:10;
>> x=0.9.^t;
>> plot (t,x)
>> hold on
>> n=0:10;
>> x=0.9.^n;
>> stem(t,x,'r')
```




```
>> t=0:0.01:10;
>> x=0.9.^t;
>> plot (t,x)
>> hold on
>> t=0:10;
>> x=0.9.^t;
>> y=0.1*round(10*x);
>> stem(t,y,'r')
>> grid on
```

n	$x(n)$	$x_q(n)$	$e_q(n)$
0	1.0000	1.0	0.0
1	0.9000	0.9000	0.0
2	0.8100	0.8000	-0.0100
3	0.7290	0.7000	-0.0290
4	0.6561	0.7000	0.0439
5	0.5905	0.6000	0.0095
6	0.5314	0.5000	-0.0314
7	0.4783	0.5000	0.0217

8 9 10	0.4305 0.3874 0.3487	0.4000 0.4000 0.3000	-0.0305 0.0126 -0.0487

Using rounding process for quantization. The other method is truncation , which discards the excess digits.

- The values allowed in the digital signal are called *quantization level*.
- Distance Δ between two quantization level is called *quantization step size* or *resolution*
- If we use rounding process the quantization error is the range of

$$-\frac{\Delta}{2} \le e_q(n) \le \frac{\Delta}{2}$$

- If $x_{\min}$ and $x_{\max}$ represent the minimum and maximum value of $x(n)$ and *L* is number of *quantization level*, then

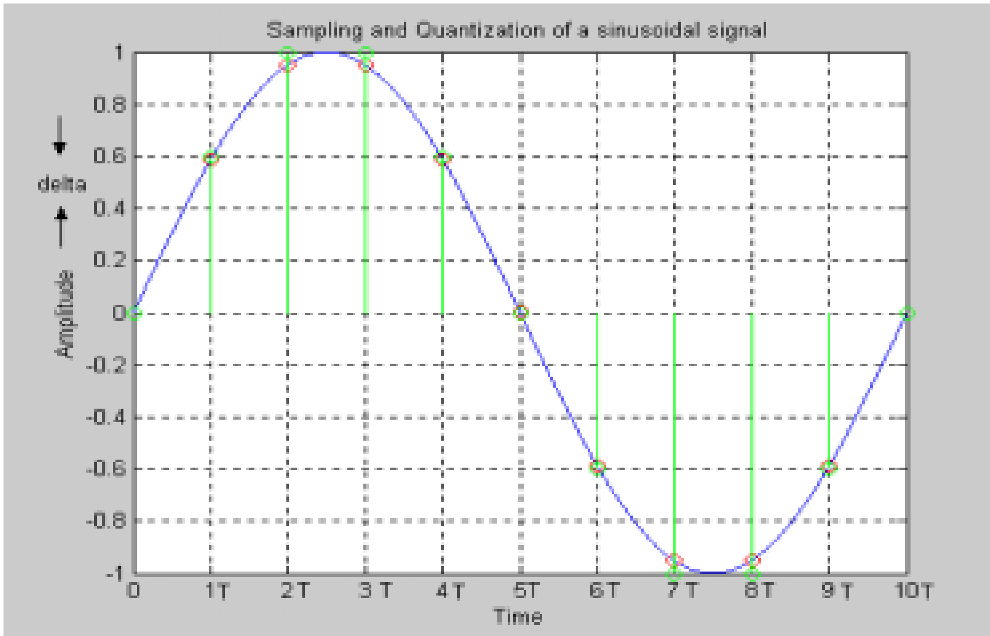
$$\Delta = \frac{x_{\max} - x_{\min}}{L-1}$$

In the example $x_{\min} = 0$, $x_{\max} = 1$, and $L = 11$, which leads to $\Delta = 0.1$.

Note:

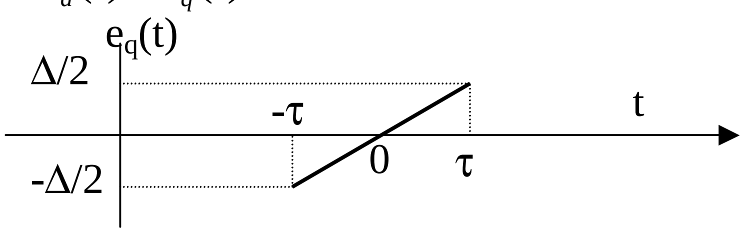
If *L* increases, Δ increases. Hence, the quantization error $e_q(n)$ decreases and the accuracy of the quantizer increases.

Quantization of Sinusoidal Signal



Let's look at the quantizer error by quantizing the analog sinusoidal signal $x_a(t)$.

The analog signal $x_a(t)$ is almost linear between quantization levels. The quantization error $e_q(t) = x_a(t) - x_q(t)$



Here $e_q(t) = \frac{\Delta}{2\tau} t$, $-\tau \le t \le \tau$

The mean-square error power P_q is

$$P_q = \frac{1}{2\tau}\int_{-\tau}^{\tau} e_q^2(t)dt = \frac{1}{\tau}\int_{0}^{\tau} e_q^2(t)dt$$

$$P_q = \frac{1}{\tau}\int_0^{\tau}\left(\frac{\Delta}{2\tau}\right)^2 t^2 dt = \frac{1}{\tau}\left(\frac{\Delta}{2\tau}\right)^2 \frac{t^3}{3}\Bigg|_0^{\tau} = \frac{\Delta^2}{12}$$

For b bit the all range is 2A, then

$$\Delta = \frac{2A}{2^b}$$

Hence, the mean-square error power P_q for the signal $x_a(t)$ is

$$P_q = \frac{A^2}{2}$$

The average power of the signal $x_a(t)$ is

$$P_x = \frac{1}{T_p}\int_0^{T_p}\left(A\cos\Omega t\right)^2 dt = \frac{A^2}{2}$$

The ratio of the signal average power to the noise power is *the signal-quantization noise ration* (SQNR) gives

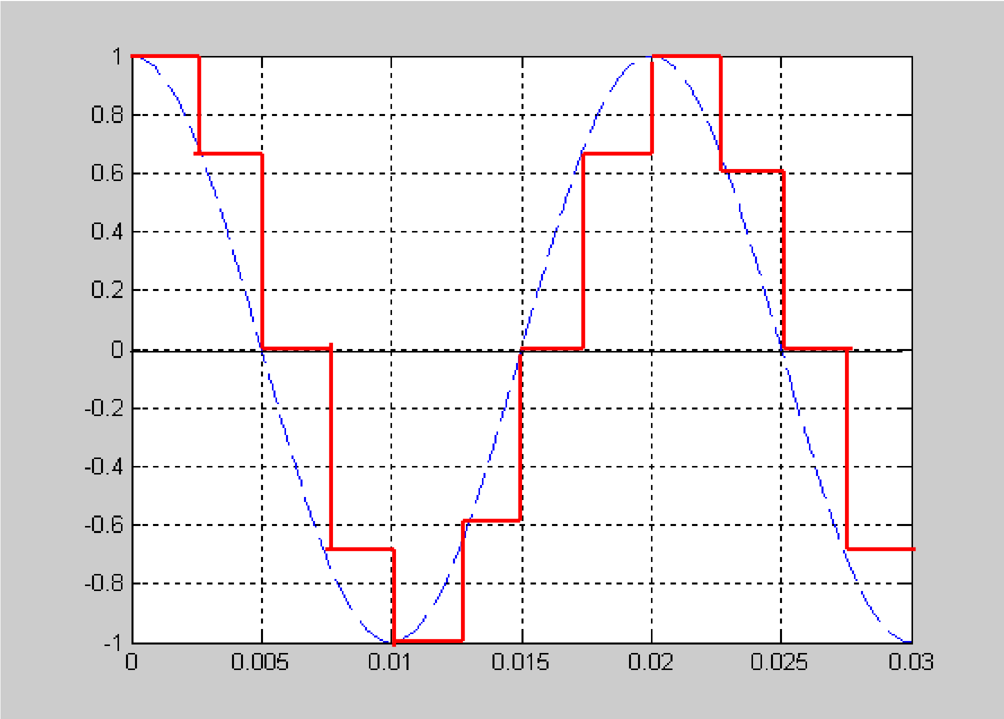
$$SQNR = \frac{P_x}{P_q} = \frac{3}{2}2^{2b}$$

In dB,

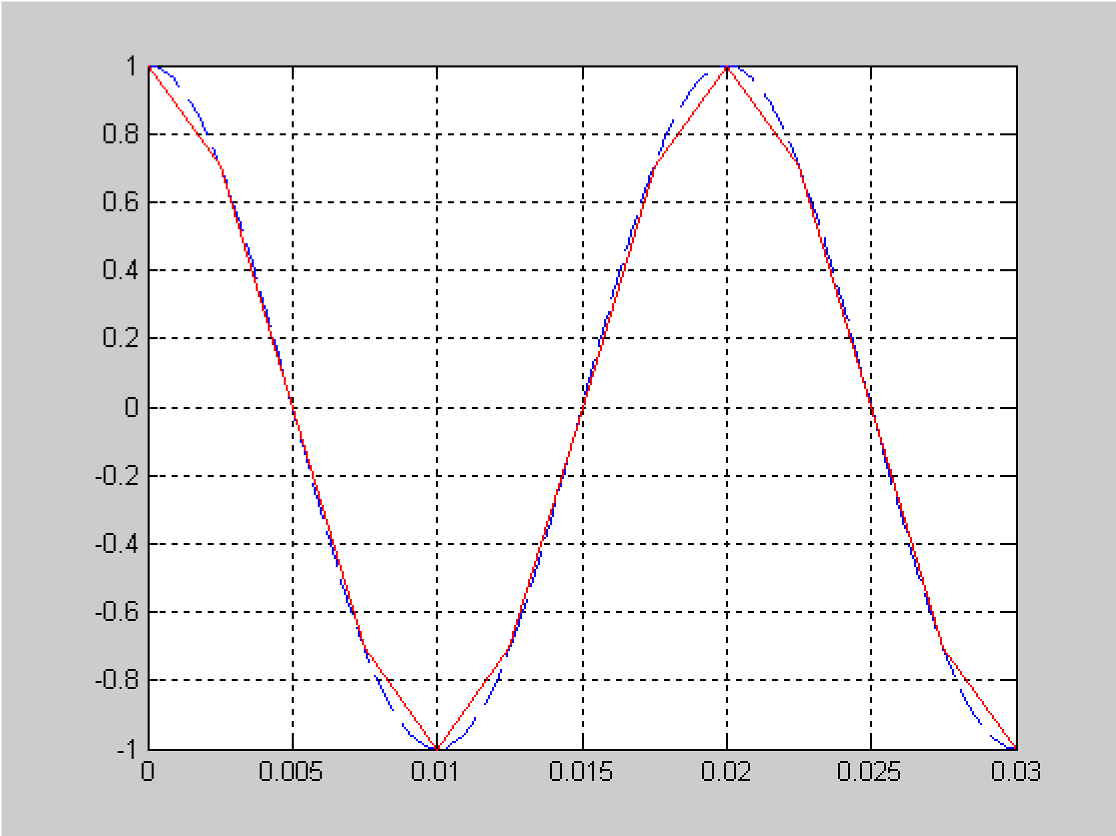
$$SQNR(dB) = 10\log_{10} SQNR = 1.76 + 6.02b$$

Digital-to-Analog Conversion

Some cases we may need to convert digital signal to analog signal again.
The process of converting a digital signal into an analog signal is called Digital-to-Analog (DAC).
All D/A converters use some kind of interpolation. A simple form of D/A conversion is zero-order hold or staircase approximation. Simply holds constant the value of one sample until the next one is received.



A Linear interpolation is connect successive samples with strait-line. It needs T second delay so that has knowledge about next sample values.



Better interpolation can be achieved by using more sophisticated high-order interpolation techniques.

Problem 1.7

An analog signal contains frequencies up to 10Khz.

a. What range of sampling frequencies allows exact reconstuction of this signal from the samples?
b. Suppose that we sample this signal with a sampling frequency Fs=8 KHz. Examine what happens to the frequency F1=5Khz.
c. Repeat part (b) for a frequency F2=9Khz.

Solution:

a. $F_{\max} = 10Khz.\ F_s \geq 2F_{\max} = 20Khz.$

a. If $F_s = 8Khz.\ F_{fold} = \frac{F_s}{2} = 4Khz.$

So, $F = 5Khz$ will be alias of 3KHz.

c. $F = 9Khz$ will be alias of 1KHz.