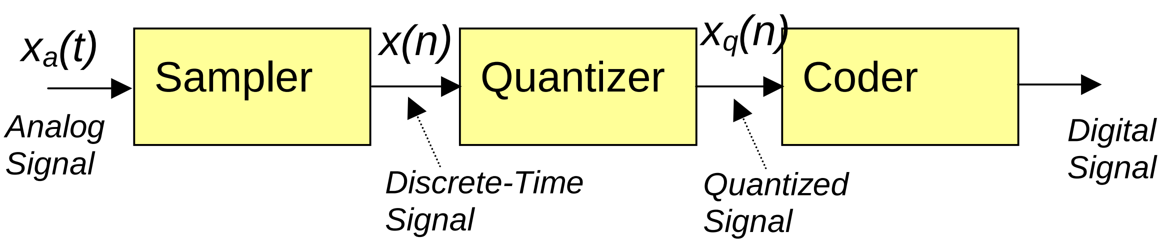
Analog-to-Digital Conversion (ADC)

In many real-world application, the signals are analog.

To process analog signal by digital, we need to convert them into digital signal
This process is called Analog-to-Digital conversion and devices are A/D Converter (ADCs).

A/D Conversion has three steps:



1. Sampling:

- Conversion of a continuous-time signal into a discrete-time signal
- Taking "samples" of the continuous-time signal at discrete-time instants.
- Sampling interval is T.

$$x(n) = x_a(nT)$$

2. Quantization:

- Conversion of a discrete-time continuous valued signal into a discrete-time, discrete valued digital signal $x_q(n)$
- Digital signal values are a finite set of possible values.

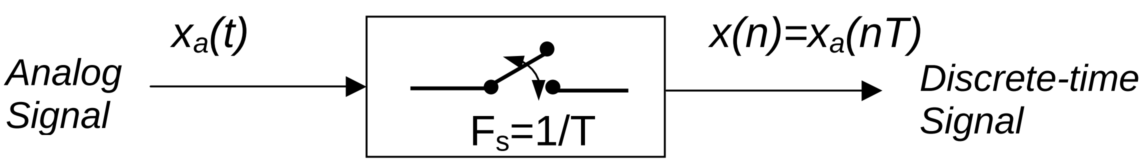
- The differences between $x_q(n)$ and $x(n)$ $(x(n) - x_q(n))$ is called the quantization error.

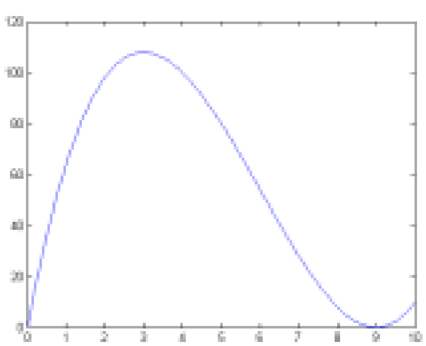
Discrete–time Signals are defined only at certain specific values of time or variable.

3. Coding

Each discrete value $x_q(n)$ is represented by a b-bit binary sequence.

Sampling of Analog Signals





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>> x=0:0.1:10;
>> y=x.^3-18*x.^2+81*x;
>> plot(x,y)
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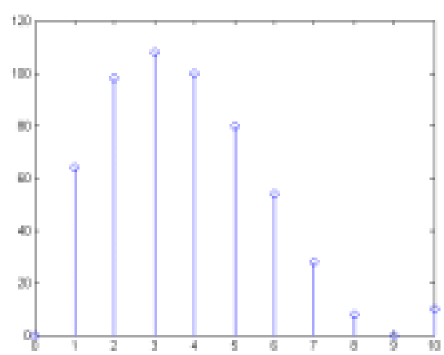


```
>> x=0:10;
>> y=x.^3-18*x.^2+81*x;
>> stem(x,y)
```

The discrete-time signal $x(n)$ is obtained by "taking-samples" of the analog signal $x_a(t)$ every T second.

$$x(n) = x_a(nT)$$

The time interval T is called the *sampling period* or *sampling interval*

The sampling rate or the sampling frequency is found as

$$F_s = \frac{1}{T} \quad [Hz]$$

The relationship between the variable t of analog signal and the variable n of discrete-time sinal is

$$t = nT = \frac{n}{F_s}$$

Consider an analog sinusoidal signal

$$x_a(t) = A\cos(2\pi Ft + \theta)$$

Sampling frequency is $F_s = 1/T$, so that

$$x(n) = x_a(nT) = A\cos(2\pi FnT + \theta)$$

$$= A\cos(2\pi F \frac{n}{F_s} + \theta)$$

or

$$x(n) = A\cos(\omega n + \theta)$$

We call relative or normalize frequency that

$$f = \frac{F}{F_s}$$

Equivalently,

$$\omega = \frac{2\pi F}{F_s} = \frac{\Omega}{F_s} = \Omega T$$

Relations between analog signals and Discrete-time signal

Continuous-time signal		Discrete-time signal
$\Omega = 2\pi F$ Ω [radians/s] F [Hz]		$\omega = 2\pi f$ ω [radians/sample] f [cycles/sample]
	$\omega = \Omega T,\ f = F/F_s$ ⟹ ⟸ $\Omega = \omega/T,\ F = f.F_s$	$-\pi \le \omega \le \pi$ $-\frac{1}{2} \le f \le \frac{1}{2}$
Range $-\infty < \Omega < \infty$ $-\infty < F < \infty$		Range $-\frac{\pi}{T} \le \Omega \le \frac{\pi}{T}$ $-\frac{F_s}{2} \le F \le \frac{F_2}{2}$

The fundamental different between analog signal and discrete-time signal is frequency range.

The highest frequency in the discrete-time signal is $\omega = \pi$ or $f = 1/2$, the sampling rate F_s, the corresponding highest value of F and Ω are

$$F_{\max} = \frac{F_s}{2} = \frac{1}{2T}$$

$$\Omega_{\max} = \pi F_s = \frac{\pi}{T}$$

Example: Consider two analog signal

$$x_1(t) = \cos 2\pi 10t$$
$$x_2(t) = \cos 2\pi 50t$$

The sampling rate is $F_s = 40\,\mathrm{Hz}$

Corresponding discrete-time signals are

$$x(n) = \cos 2\pi\left(\frac{10}{40}\right)n = \cos\frac{\pi}{2}n$$
$$x(n) = \cos 2\pi\left(\frac{50}{40}\right)n = \cos\frac{5\pi}{2}n$$

We know that

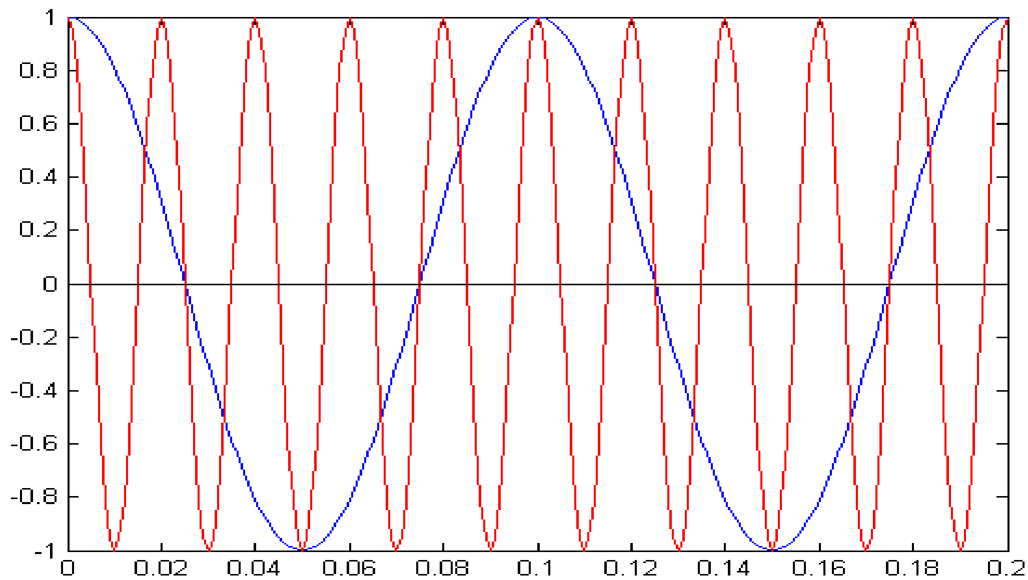
$$\cos\frac{5\pi}{2}n = \cos\left(2\pi + \frac{\pi}{2}\right)n = \cos\frac{\pi}{2}n.$$

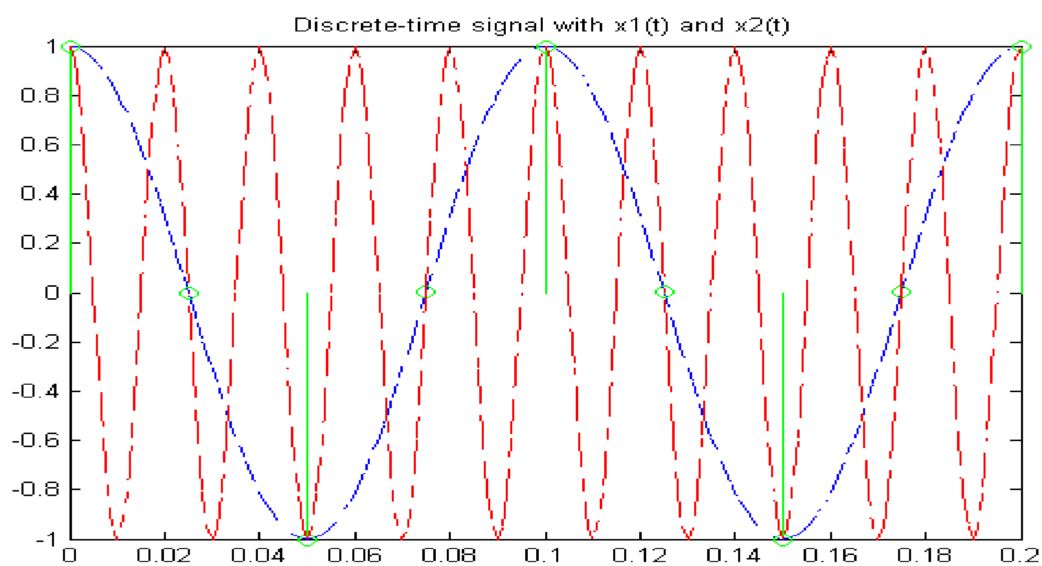
Hence

$$x_1(n) = x_2(n)$$

The frequency $F_2 = 50\,\mathrm{Hz}$ is an alias of the frequency $F_1 = 10\,\mathrm{Hz}$ at the sampling rate of $F_s = 40\,Hz$. Even $F_k = (F_1 + 40k),\ k = 1, 2, 3, ...$ are alias of F_1 at the sampling rate at $F_s = 40\,Hz$.

In general form, $F_k = (F_0 + kF_s),\ k = \pm 1, \pm 2, \pm 3, ...$are creates an alias for frequency F_0 of analog signal, which are outside of frequency range $-\frac{F_s}{2} \le F \le \frac{F_2}{2}$.



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>> t=0:0.001:0.2;
>> x1=cos(2*pi*10*t);
>> plot (t,x1,'--')
>> hold on
>> x2=cos(2*pi*50*t);
>> plot (t,x2,'r--')
>> n=0:0.025:0.2;
>> y1=cos(2*pi*10*n);
>> stem (t,y1,'g-')
>> Title('Discrete-time signal with x1(t) and x2(t)')
```

Example:

$$x_a(t) = 3\cos 100\pi t$$

a. Find the minimum sampling rate required to avoid aliasing.
b. If $F_s = 200\ Hz$, What is the discrete-time signal after sampling?
c. If $F_s = 75 Hz$, What is the discrete-time signal after sampling?
d. What is the frequency F of a sinusoidal that yields sampling identical to obtained in part c?

a. $\Omega = 100\pi \rightarrow F = 50\ \text{Hz}$

The minimum sampling rate is

$$F_s = 2F = 100\,\text{Hz}$$

and the discrete-time signal is

$$x(n) = 3\cos\frac{100\pi}{100}n = 3\cos\pi n$$

b. If $F_s = 200\ Hz$, the discrete-time signal is

$$x(n) = 3\cos\frac{100\pi}{200}n = 3\cos\frac{\pi}{2}n$$

c. If $F_s = 75\ \text{Hz}$, the discrete-time signal is

$$x(n) = 3\cos\frac{100\pi}{75}n = 3\cos\frac{4\pi}{3}n$$

$$= 3\cos\left(2\pi - \frac{2\pi}{3}\right)n$$

$$= 3\cos\frac{2\pi}{3}n$$

d. For the sampling rate $F_s = 75 \text{ Hz}$,

$F = fF_s = f75$, and $f = \frac{1}{3}$ in part in (c). Hence

$$F = \frac{75}{3} = 25\,\text{Hz}$$

So, the analog sinusoidal signal is

$$\begin{aligned} y_a(t) &= 3\cos 2\pi F t \\ &= 3\cos 50\pi t \end{aligned}$$

The Sampling Theorem

We must have some information about the analog signal especially the frequency content of the signal, to select the sampling period T or sampling rate F_s .

For example: A speech signal goes below around 3Khz.
A TV signal is up to 5Mhz.

Any analog signal can be represented as sum of sinusoids of different amplitudes, frequencies, and phases.

$$x_a(t) = \sum_{i=1}^{N} A_i \cos(2\pi F_i t + \theta_i)$$

where N the number of frequency components. Suppose that Nth frequency do not exceed the largest frequency $F_{\max}$

$$|F_i| < F_{\max}$$

To avoid the aliasing problem, F_s is selected so that

$$F_s > 2F_{\max}$$

The analog signal should be in the range of

$$-\frac{1}{2} \leq f_i = \frac{F_i}{F_s} \leq \frac{1}{2}$$

or in radians

$$-\pi \leq \omega_i = 2\pi f_i \leq \pi$$

The sampling rate $F_N = 2F_{\max}$ is called the *Nyquist rate*.

Example:

Consider an analog signal

$$x_a(t) = 3\cos 50\pi t + 10\cos 300\pi t - 3\cos 100\pi t$$

The frequencies in the analog signal

$F_1 = 25\,\text{Hz}$, $F_2 = 150\,\text{Hz}$, $F_3 = 50\,\text{Hz}$

The largest frequency is

$F_{\max} = F_2 = 150\,\text{Hz}$

The Nyquist rate is

$F_N = 2F_{\max} = 300\ \text{Hz}$