Continuous-Time Versus Discrete-Time Signals

Continuous time signals (Analog Signal): The signal can be described by functions of a continuous time or variable

Example:

$$x_1(t) = A\sin 4\pi t$$

$$x_2(t) = e^{-|t|}, \text{ for } \infty < t < \infty$$

Discrete–time Signals are defined only at certain specific values of time or variable.

Example;

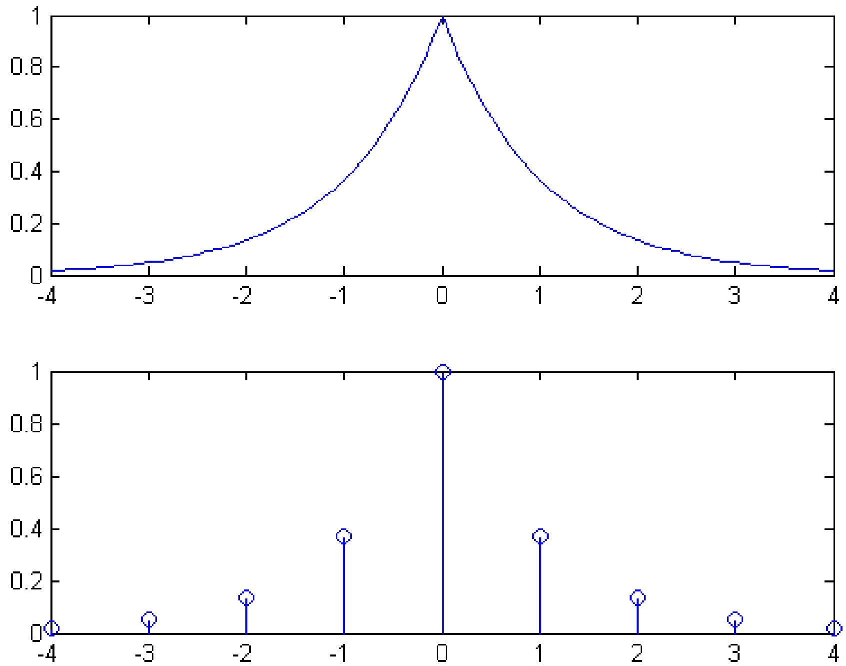
$$x(t_n) = e^{-|t_n|}, \text{ for } n = 0, \pm 1, \pm 2, \ldots$$

where index n is discrete-time instants as the independent variable.
A discrete-time signal can be represented by a sequence of real or complex number.
If the time instants t_n are equally spaced

$$t_n = nT$$

We can represent a discrete-time signal

$$x(nT) = e^{-|nT|}, \text{ for } n = 0, \pm 1, \pm 2, \ldots$$




```
>> t=-4:0.1:4;
>> x=exp(-abs(t));
>> subplot(2,1,1),plot(t,x)
>> t=-4:1:4;
>> x=exp(-abs(t));
>> subplot(2,1,2),stem(t,x,'-')
```

By selecting values of an analog signal at discrete-time instants is called *sampling process*

Another example:
Let's have an analog signal

$$x(t) = 0.8^t,\ t \geq 0$$

By sampling the analog signal, we have

$$x(n) = \begin{cases} 0.8^n, & \text{if } n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

All measurements are at a regular interval of time in discrete-time signal.
There are some signals that are in discrete-time. Such as, counting how many people enter a store every hour.

Continuous-Valued Versus Discrete-Valued Signal

The values of continuous-time signal are continuous if A signal takes on all possible values on a finite or infinite range.
The values of discrete-time signal are discrete if a signal takes on values from a finite set of possible values.

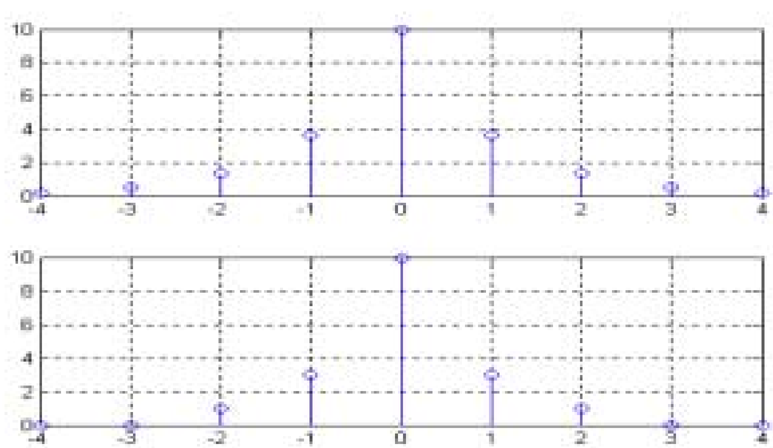
A discrete-time signal having a set of discrete values is called a ***digital signal***

If the signal is converted from analog to digital,

- a analog signal needs to be sampled at discrete instants in time to obtain a discrete signal.
- Then, it needs to be *quantization* the values to set of discrete values. It may need some rounding or truncation.

Example: If the digital-signal values are integer and between 0 and 15, the discrete values signal is quantized into these integer value.

Discrete-value : 5.7 will be 5 by truncation or 6 by rounding




```
>> t=-4:1:4;
>> x=10*exp(-abs(t));
>> subplot(2,1,1),stem(t,x,'-')
>> grid on
>> y=floor(10*exp(-abs(t)));
>> subplot(2,1,2),stem(t,y,'-')
>> grid on
```

Deterministic Versus Random Signal

- Any signal that can be uniquely described by an explicit mathematical expression, a table of data, or a well-defined rule is called deterministic.
- There are some signals that either cannot be described to any reasonable degree of accuracy by explicit mathematical formulas, or such a description is complicated to be of any practical use. We called these signal as random
- We use statistical techniques instead of explicit formulas. We call this in math probability and stochastic process to provides information for theoretical analysis of random-signal

The Concept of Frequency in Continuous-Time and Discrete-Time Signal

a. Continuous Time Signal

Let's have the following continuous-time sinusoidal signal:

$$x_a(t) = A\cos(\Omega t + \theta), -\infty < t < \infty$$

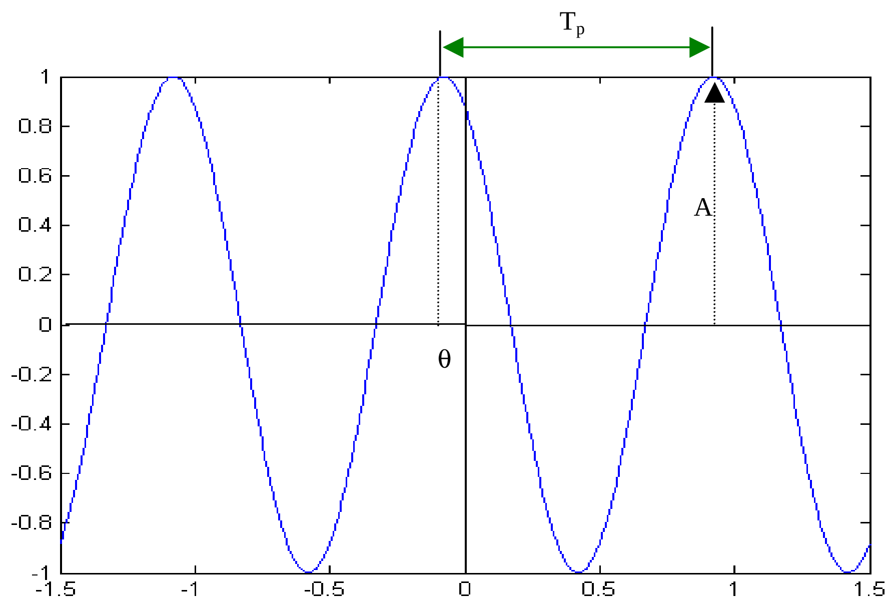
where

A : the amplitude of the signal
Ω : the frequency in radians per second
θ : the phase in radians

The frequency can be expressed in cycles/s or Hertz(Hz)

$$F = \frac{\Omega}{2\pi}$$

The period is define as $T_p = \frac{1}{F}$



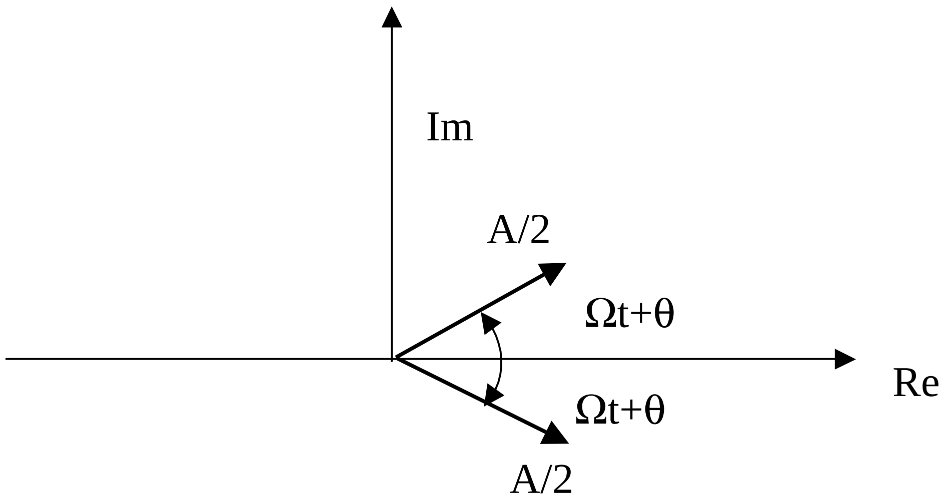
- The analog sinusoidal signal can repeat every period

$$x_a(t+T_p) = x_a(t)$$

- Increasing the frequency means decreasing the period of the signal, so that increase the rate of oscillation of the signal

The analog sinusoidal signal can be expressed in complex exponent for as

$$x_a(t) = A\sin(\Omega t + \theta) = \frac{A}{2}e^{j(\Omega t+\theta)} + \frac{A}{2}e^{-j(\Omega t+\theta)}$$



b. Discrete-Time Sinusoidal Signal

A dicrete-time sinusoidal signal may be expressed as

$$x(n) = A\cos(\omega n + \theta), -\infty < n < \infty$$

where n : integer variable
A : the amplitude of the signal
ω : the frequency in radians per sample
θ : the phase in radians

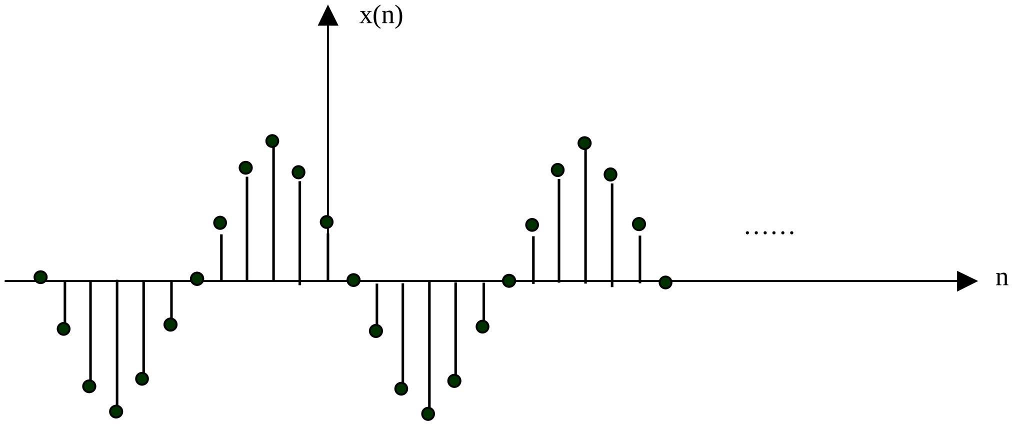
The frequency can be expressed in cycles per sample

$$f = \frac{\omega}{2\pi}$$

and the signal is

$$x(n) = A\cos(2\pi fn + \theta), -\infty < n < \infty$$

Example: a sinusoidal with the amplitude A, frequency $\omega=\pi/6$ radians per sample (f=1/12) and phase $\theta=\pi/3$



- A discrete-time sinusoidal is periodic only if its frequency is rational number

$$x(n+N) = x(n)$$

$$\cos[2\pi f_0(N+n)+\theta] = \cos[2\pi f_0 n+\theta]$$

It is true if and only if

$$2\pi f_0 N = 2k\pi \quad \text{or} \quad f_0 = k/N$$

- Discrete-time sinusoids whose frequencies are separated by an integer multiple of 2π are identical

$$\cos[(\omega_0+2\pi)n+\theta] = \cos(\omega_0 n+2\pi n+\theta) = \cos(\omega_0 n+\theta)$$

$$x(n) = A\cos[(\omega_0+2k\pi)n+\theta],\ \ for\, k = 0,1,2...$$

are identical

- The highest rate of oscillation in a discrete-time sinusoidal is attained when $\omega=\pi$ (or $\omega=-\pi$) equivalent to f=1/2 (or f=-1/2)

For $x(n) = A\cos\omega_0 n$

ω	0	π/8	π/4	π/2	π
f	0	1/16	1/8	1/4	1/2
N	∞	16	8	4	2

If $\pi \le \omega_0 \le 2\pi$, it creates an aliasing. How?
Let's consider
$\omega_1 = \omega_0$, which $\pi \le \omega_0 \le 2\pi$ and
$\omega_2 = 2\pi - \omega_0$, which $\pi \le \omega_0 \le 2\pi$

$$x_1(n) = A\cos\omega_1 n = A\cos\omega_0 n$$
$$\begin{aligned} x_2(n) = A\cos\omega_2 n &= A\cos(2\pi - \omega_0)n \\ &= A\cos\omega_0 n \\ &= x_1(n) \end{aligned}$$

Hence, ω_2 is an alias of ω_1.