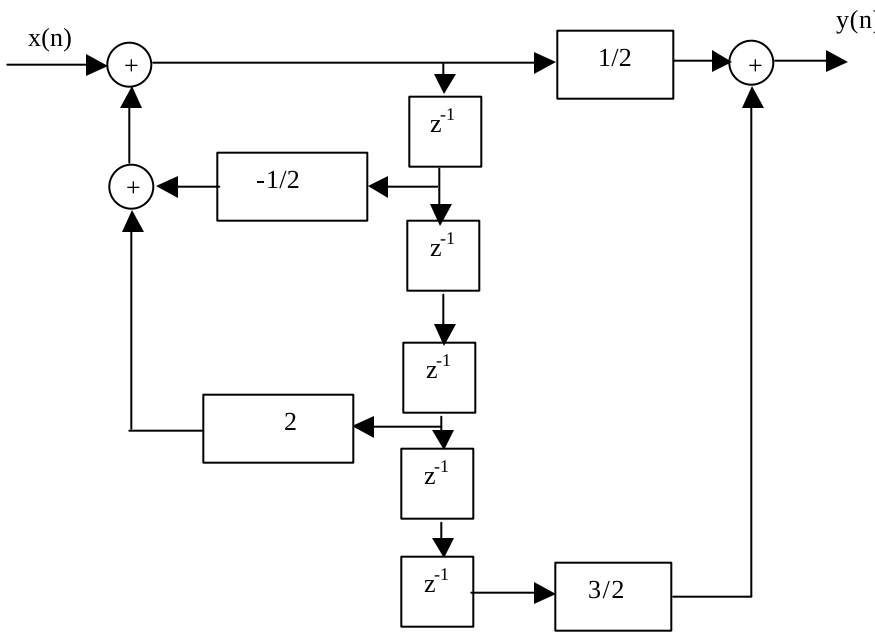
Problem Solutions

Chapter .2

<u>43 a</u> Determine the direct form II realization for the following LTI system

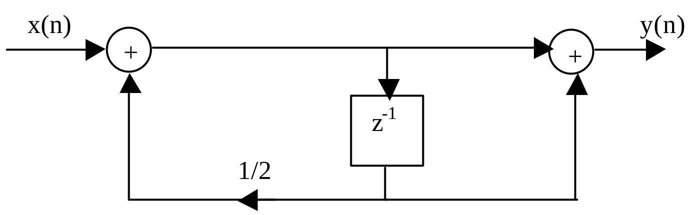
$$2y(n)+y(n-1)-4y(n-3)=x(n)+3x(n-5)$$

$$y(n)+\frac{1}{2}y(n-1)-2y(n-3)=\frac{1}{2}x(n)+\frac{3}{2}x(n-5)$$



HW.# 3
2.43.b

<u>2.44</u>



HW.# 3
2.47

a. Compute the first samples of its impulse response

$$x(n)=d(n)$$

$$x(n)=\{1,0,0,\ldots\}$$
$$\uparrow$$

$$y(n)=\frac{1}{2}y(n-1)+x(n)+x(n-1)$$

$$y(0)=x(0)=1$$

$$y(1)=\frac{1}{2}y(0)+x(1)+x(0)=\frac{3}{2}$$

$$y(2)=\frac{1}{2}y(1)+x(2)+x(1)=\frac{3}{4}$$
$$y(3)=\frac{1}{2}y(2)+x(3)+x(2)=\frac{3}{8}$$
$$y(n)=\left\{1,\frac{3}{2},\frac{3}{4},\frac{3}{8},\frac{3}{16},\frac{3}{32},\frac{3}{64},\ldots\right\}$$

b. Find the input output relation

$$y(n)=\frac{1}{2}y(n-1)+x(n)+x(n-1)$$

c. The input

$$x(n)=\{1,1,1,\ldots\}$$
$$\uparrow$$
$$y(n)=\frac{1}{2}y(n-1)+x(n)+x(n-1)$$
$$y(0)=x(0)=1$$
$$y(1)=\frac{1}{2}y(0)+x(1)+x(0)=\frac{5}{2}$$

$$y(2)=\frac{1}{2}y(1)+x(2)+x(1)=\frac{13}{4}$$
$$y(3)=\frac{1}{2}y(2)+x(3)+x(2)=\frac{29}{8}$$

.....

d. Use convolution

$$y(n)=u(n0*h(n)=\sum_{k=0}^{\infty}u(k)h(n-k)=\sum_{k=0}^{n}h(n-k)$$
$$y(0)=h(0)=1$$
$$y(1)=h(0)+h(1)=\frac{5}{2}$$
$$y(2)=h(0)+h(1)+h(2)=\frac{13}{4}$$
$$y(3)=h(0)+h(1)+h(2)+h(3)=\frac{29}{8}$$

2.54 Find the y(n) for the following equation

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

$$x(n) = (-1)^n u(n)$$

The characteristic equation

$$l^2 - 4l + 4 = 0, \rightarrow l_1 = 2, \quad l_2 = 2$$

The homogenous solution is

$$y_h(n) = c_1 2^n + c_2 n 2^n$$

The particular solution is

$$y_p(n) = k(-1)^n u(n)$$

$$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For n=2

$$k + 4k + 4k = 1 + 1 \quad \rightarrow k = \frac{2}{9}$$

The total solution is

$$y(n) = y_h(n) + y_p(n) = \left[c_1 2^n + c_1 n 2^n + \frac{2}{9}(-1)^n \right] u(n)$$

Using the initial condition,

$$y(-1) = y(-2) = 0$$

we can obtain from difference equation at

n=0
$$y(0) - 4y(-1) + 4y(-2) = x(0) - x(-1)$$
$$y(0) = 1$$

n=1
$$y(1) - 4y(0) + 4y(-1) = x(1) - x(0)$$
$$y(1) = 2$$

From the total solution

$$y(0) = c_1 + \frac{2}{9} = 1$$

$$y(1) = c_1 2 + c_2 2 - \frac{2}{9} = 2$$

$$c_1 = \frac{7}{9}, \quad c_1 = \frac{1}{3}$$

$$y(n) = \left[\frac{7}{9} 2^n + \frac{1}{3} n 2^n + \frac{2}{9}(-1)^n \right] u(n)$$

2.55 Find the impulse response h(n) for the following equation

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

The homogenous solution is

$$y_h(n) = c_1 2^n + c_2 n 2^n$$

So system response

$$h(n) = \left[c_1 2^n + c_2 n 2^n\right] u(n)$$

To find the constant

At n=0 $y(0) - 4y(-1) + 4y(-2) = d(0) - d(-1)$

$y(0) = 1$

At n=1 $y(1) - 4y(0) + 4y(-1) = d(1) - d(0)$

$y(1) = 3$

From the system response h(n)

$$y(0) = c_1 = 1$$

$$y(1) = c_1 2 + c_1 2 = 3, \quad c_2 = \frac{1}{2}$$

$$h(n) = \left[2^n + \frac{1}{2} n 2^n\right] u(n)$$

2.59

Find the autocorrelation sequence of the following signals

a. $x(n) = \{\underset{\uparrow}{1}, 2, 1, 1\}$

b. $y(n) = \{\underset{\uparrow}{1}, 1, 2, 1\}$

HW.# 3
2.58

a.

$$g_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) x(n-l)$$

$$g_{xx}(-3) = x(0)x(3) = 1$$
$$g_{xx}(-2) = x(0)x(2) + x(1)x(3) = 3$$
$$g_{xx}(-1) = x(0)x(1) + x(1)x(2) + x(2)x(3) = 5$$
$$g_{xx}(0) = x(0)x(0) + x(1)x(1) + x(2)x(2) + x(3)x(3) = 7$$
$$g_{xx}(1) = x(1)x(0) + x(2)x(1) + x(3)x(2) = 5$$

….

$$g_{xx}(l) = \{1, 3, 5, 7, 5, 3, 1\}$$

Extra HW
One of the following of
2.63, 2.64