2.3.5 Causal LTI Systems

Causal system is the output is depends only on present and past input signal

The output for LTI systems is given by

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

For causal system ($h(n) = 0 \text{ for } n < 0$), the output becomes

$$y(n) = \sum_{k=0}^{\infty} h(k)x(n-k)$$

$$= \sum_{k=-\infty}^{n} x(k)h(n-k)$$

If the input is a causal sequence $(x(n) = 0 \text{ for } n < 0)$ too, convolution becomes

$$y(n) = \sum_{k=0}^{n} x(k)h(n-k) = \sum_{k=0}^{n} h(k)x(n-k)$$

2.3.1 Stability of LTI Systems

An arbitrary relaxed system is a BIBO stable if only if its output sequence y(n) is bounded for every bounded input x(n).

$|x(n)| = M_x < \infty$ and $|y(n)| = M_y < \infty$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

For a BIBO system

$$|y(n)| = \left| \sum_{k=-\infty}^{\infty} h(k)x(n-k) \right|$$

$$\leq \sum_{k=-\infty}^{\infty} |h(k)||x(n-k)|$$

$$\leq M_x \sum_{k=-\infty}^{\infty} |h(k)|$$

Hence

$$S_h = \sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

A LTI system is stable if its impulse response is absolutely summable.

2.3.7 System with Finite-Duration and Infinite-Duration Impulse Response

Finite-Duration Impulse Response (FIR) system has an impulse response that is zero outside of some finite time interval.

$$h(n) = 0, \; n < 0 \text{ and } n \geq M$$

The convolution is

$$y(n) = \sum_{k=0}^{M-1} h(k)x(n-k)$$

FIR system has a finite memory of length-M samples.

Infinite-Duration Impulse Response (IIR) of LTI has an infinite-duration impulse response that is not finite time interval.

The convolution is

$$y(n)=\sum_{k=0}^{\infty}h(k)x(n-k)$$

where causality has been assumed.

2.4 DISCRETE-TIME SYSTEMS DECRIBED BY DIFFERENCE EQUATIONS

2.4.1 Recursive and Nonrecursive Discrete-Time Systems

Let's have a DTS that gives the cumulative average

$$y(n)=\frac{1}{n+1}\sum_{k=0}^{n}x(k)$$

It requires of all the input samples x(k) for $0\le k\le n$. If n is large needs more memory requirement.

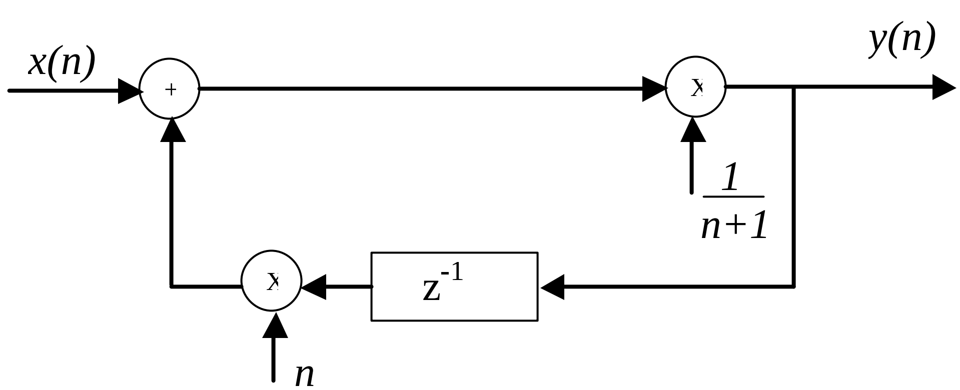
We can find y(n) more efficient by utilizing the output y(n-1).

$$y(n-1)=\frac{1}{n}\sum_{k=0}^{n-1}x(k)$$

$$y(n)=\frac{1}{n+1}\sum_{k=0}^{n-1}x(k)+x(n)$$

$$=\frac{1}{n+1}\left(ny(n-1)+x(n)\right)$$

$$=\frac{n}{n+1}y(n-1)+\frac{1}{n+1}x(n)$$



This system requires two multiplication, one addition, and one memory location. This is a recursive system which means the output at time n depends on any number of a past output values. So, a recursive system has feed back output of the system into the input. This feed back loop contains a delay element.

Here

$$y(0) = x(0)$$
$$y(1) = \frac{1}{2} y(0) + \frac{1}{2} x(1)$$
$$y(2) = \frac{2}{3} y(1) + \frac{1}{3} x(2)$$
$$\cdots$$
$$y(n_0) = \frac{n_0}{n_0 + 1} y(n_0 - 1) + \frac{1}{n_0 + 1} x(n_0)$$

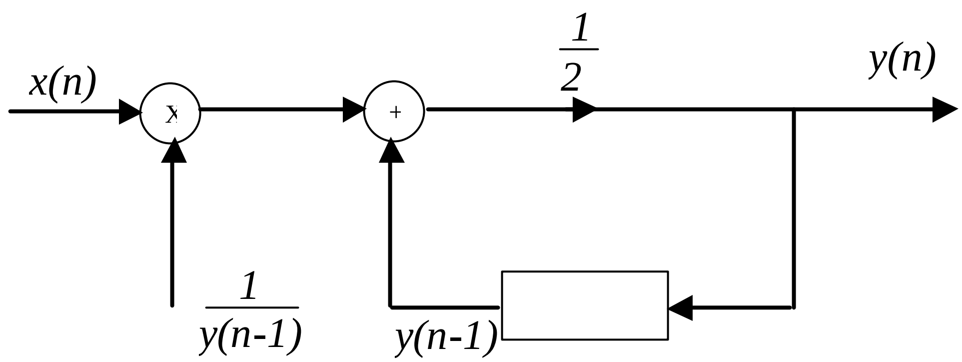
Example:
Compute the square root of a positive number A using the iteration algorithm

$$s_n = \frac{1}{2}\left(s_{n-1} + \frac{A}{s_{n-1}} \right), \quad n = 0,1,...$$

where s_{-1} is initial guess of $\sqrt{A}$

Consider a recursive system

$$y(n) = \frac{1}{2}\left(y(n-1) + \frac{x(n)}{y(n-1)} \right)$$



If y(n) depends only on the present and past input, it is called nonrecursive.
For the causal FIR systems

$$y(n) = \sum_{k=0}^{M} h(k)x(n-k)$$
$$= h(0)x(n) + h(1)x(n-1) + ... + h(M)x(n-M)$$
$$= F\left[x(n), x(n-1), ..., x(n-M)\right]$$