

Frequency Response of a Circuit

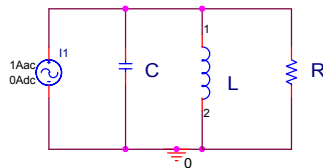
Parallel Resonance circuit

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Frequency Response of a Circuit

Parallel Resonance circuit



$$Y = \frac{1}{R} + j(\omega C - \frac{1}{\omega L})$$

$$\frac{I_R(j\omega)}{I(j\omega)} = \frac{V(j\omega)}{V(j\omega)Y} = \frac{1}{1 + j(\omega CR - \frac{R}{\omega L})}$$

To find frequency response, substitute $s=j\omega$ in equation

Magnitude Response

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega CR - \frac{R}{\omega L})^2}}$$

Phase Response

$$\theta(j\omega) = -\tan^{-1}\left(\omega CR - \frac{R}{\omega L}\right)$$

Frequency Response of a Circuit

Parallel Resonance circuit

At resonance frequency, the transfer function will be real.
Or system total impedance will be real

$$j\omega_0 C + \frac{1}{j\omega_0 L} = 0$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

H_{\max} will be at $|H(j\omega_0)|$

substitute $\omega_0 = \frac{1}{\sqrt{LC}}$

$$|H(j\omega_0)| = \frac{1}{\sqrt{1 + (\omega_0 CR - \frac{R}{\omega_0 L})^2}}$$

Result

$$|H(j\omega_0)| = \frac{1}{\sqrt{1 + (\frac{1}{\sqrt{LC}} CR - \frac{\sqrt{LC} R}{L})^2}} = 1$$

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Set $(1/\sqrt{2})H_{\max}$ to find cutoff frequencies

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + (\omega_c CR - \frac{R}{\omega_c L})^2}}$$

Result

$$\omega_c^2 + \frac{1}{RC}\omega_c - \frac{1}{LC} = 0$$

$$\omega_{c1} = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \left(\frac{1}{LC}\right)}$$

Confirm

$$\omega_c^2 - \frac{1}{RC}\omega_c - \frac{1}{LC} = 0$$

$$\omega_{c2} = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \left(\frac{1}{LC}\right)}$$

$$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{\frac{1}{LC}}$$

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The Bandwidth β is

$$\begin{aligned} \beta &= \omega_{c2} - \omega_{c1} \\ &= \left[\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \left(\frac{1}{LC}\right)} \right] - \left[-\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \left(\frac{1}{LC}\right)} \right] \quad \beta = \frac{1}{RC} \end{aligned}$$

The Quality factor Q is

$$Q = \frac{\omega_0}{\beta} = \omega_0 RC = \frac{1}{\sqrt{LC}} RC$$

$$Q = \sqrt{\frac{C}{L}} R$$

$$Q = \frac{f_0}{f_{c2} - f_{c1}}$$

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The cutoff frequencies in terms of β is ω_0

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + (\omega_0)^2}$$

$$\omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + (\omega_0)^2}$$

The cutoff frequencies in terms of Q is ω_0

$$\omega_{c1} = \omega_0 \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right]$$

$$\omega_{c2} = \omega_0 \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \right]$$

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Frequency Response of a Circuit

Example Using parallel RLC circuit, design band pass filter that bandwidth 750 Hz and a center frequency of 1Khz Hz. Use a 10 mH inductor, Find R,C, ω_{c1} , ω_{c2} ,and Q

Let's find L first.

$$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{\frac{1}{LC}}$$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(2\pi 1000)^2 10 \times 10^{-3}} = 2.53 \mu F$$

Calculate R

$$R = \frac{1}{C\beta} = \frac{1}{2\pi 750(2.53)10^{-6}} = 83.75 \Omega$$

The cutoff frequencies

The Quality factor is

$$Q = \frac{\omega_0}{\beta} = 1.33$$

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + (\omega_0)^2} = 692.9 \text{ rad/s}$$

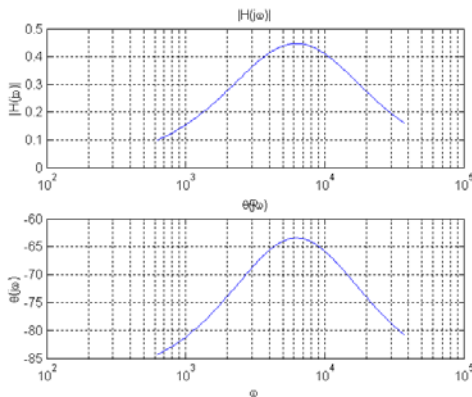
$$\omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + (\omega_0)^2} = 1443 \text{ rad/s}$$

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R=83.75 Ω , L=10 mH, C=2.53 μF , Plot F=100 – 6 KHz.

$$H(j\omega) = \frac{1}{1 + j\left(\omega CR - \frac{R}{\omega L}\right)}$$



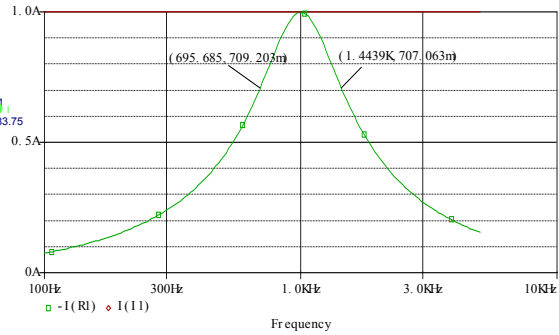
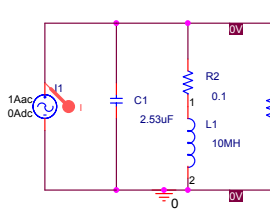
```
>> R=83.75;
>> L=0.01;
>> C=2.5330e-006;
>> f=100:10:6000;
>> w=2*pi*f;
>>
>> h=abs(1./(1+j*((w*C*R)+R./(w*L))));
>> subplot(2,1,1)
>> semilogx(w,h)
>> grid on
>> title('|H(j\omega)|')
>> xlabel('\omega')
>> ylabel('|H(j\omega)|')
>> subplot(2,1,2)
>>
>> theta=angle(1./(1+j*((w*C*R)+R./(w*L))));
>> degree=theta*180/pi;
>> semilogx(w,degree)
>> grid on
>> title('\theta(j\omega)')
>> xlabel('\omega')
>> ylabel('\theta(j\omega)')
```

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Frequency Response of a Circuit

Example

OrCad Capture

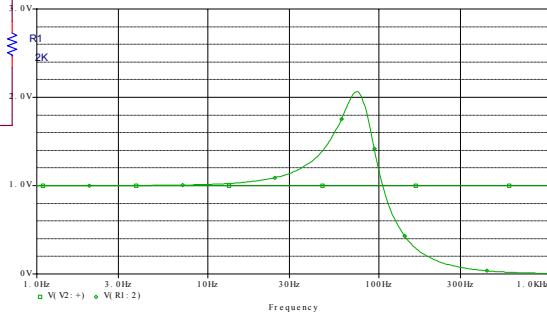
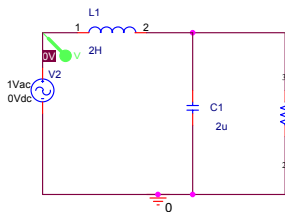


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Frequency Response of a Circuit

Example

$R=2\text{ K}\Omega$, $L=2\text{ H}$, $C=2\text{ }\mu\text{F}$, Plot $F=1\text{ Hz} - 1\text{ KHz}$.



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