Frequency Response of a Circuit

Band-Reject Filter

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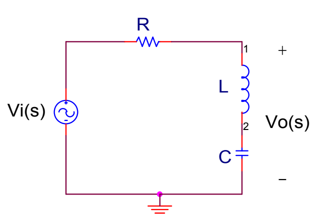
Band-Reject Filter

A Serial RLC Circuit



$$\frac{V_0(s)}{V_i(s)} = \frac{sL + \frac{1}{sC}}{sL + R + \frac{1}{sC}}$$

$$H(s) = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

To find frequency response, substitute s=jω in equation

$$H(j\omega) = \frac{-\omega^2 + \frac{1}{LC}}{-\omega^2 + \frac{R}{L}j\omega + \frac{1}{LC}}$$

Magnitude Response

$$|H(j\omega)| = \frac{\left|\frac{1}{LC} - \omega^2\right|}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{R}{L}\omega\right)^2}}$$

Phase Response

$$\theta(j\omega) = -\tan^{-1}\left(\frac{\omega\frac{R}{L}}{\frac{1}{LC} - \omega^2}\right)$$

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At resonance frequency, the transfer function will be real.
Or system total impedance will be real

$$j\omega_0 L + \frac{1}{j\omega_0 C} = 0$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

H_{min} will be at $|H(j\omega_0)|$

$$|H(j\omega_0)| = \frac{\left|\frac{1}{LC} - \omega_0^2\right|}{\sqrt{\left(\frac{1}{LC} - \omega_0^2\right)^2 + \left(\frac{R}{L}\omega_0\right)^2}}$$

substitute $\omega_0 = \sqrt{1/LC}$

Result

$$|H(j\omega_0)| = \frac{\left|\frac{1}{LC} - \frac{1}{LC}\right|}{\sqrt{\left(\frac{1}{LC} - \frac{1}{LC}\right)^2 + \left(\frac{R}{L}\sqrt{\frac{1}{LC}}\right)^2}} = 0$$

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Set $(1/\sqrt{2})H_{max}$ to find cutoff frequencies

$$\frac{1}{\sqrt{2}} = \frac{\left|\frac{1}{LC} - \omega_c^2\right|}{\sqrt{\left(\frac{1}{LC} - \omega_c^2\right)^2 + \left(\frac{R}{L}\omega_c\right)^2}}$$

$$\frac{1}{2}\left[\left(\frac{1}{LC} - \omega_c^2\right)^2 + \left(\frac{R}{L}\omega_c\right)^2\right] = \left(\frac{1}{LC} - \omega_c^2\right)^2$$

$$\left(\frac{1}{LC} - \omega_c^2\right)^2 = \left(\frac{R}{L}\omega_c\right)^2$$

$$\left(\frac{1}{LC} - \omega_c^2\right) = \left(\frac{R}{L}\omega_c\right)$$

Result

$$\omega_c^2 + \frac{R}{L}\omega_c - \frac{1}{LC} = 0 \Rightarrow \omega_{c1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

$$\omega_c^2 - \frac{R}{L}\omega_c - \frac{1}{LC} = 0 \Rightarrow \omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

Confirm

$$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{\frac{1}{LC}}$$

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The Bandwidth β is

$$\beta = \omega_{c2} - \omega_{c1}$$
$$= \left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}\right] - \left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}\right]$$

$$\beta = \frac{R}{L}$$

The Quality factor Q is

$$Q = \frac{\omega_0}{\beta} = \frac{\sqrt{\frac{1}{LC}}}{\frac{R}{L}}$$

$$Q = \sqrt{\frac{L}{CR^2}}$$

$$Q = \frac{f_0}{f_{c2} - f_{c1}}$$

Frequency Response of a Circuit

A Serial RLC Circuit

The cutoff frequencies in terms of β is ω_0

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + (\omega_0)^2}$$

$$\omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + (\omega_0)^2}$$

The cutoff frequencies in terms of β is ω_0

$$\omega_{c1} = \omega_0\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

$$\omega_{c1} = \omega_0\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

Frequency Response of a Circuit

Example: Using serial RLC circuit, design band reject filter that bandwidth 250 Hz and a center frequency of 750 Hz. Use a 100nF capacitor, Find R,L,ω_{c1}, ω_{c2},and Q

Let's find L first.

$$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{\frac{1}{LC}}$$

$$L = \frac{1}{\omega_0^2 C} = \frac{1}{(2\pi 750)^2 100x10^{-9}} = 450\,mH$$

Calculate R

$$R = \beta L = 2\pi 750(450)10^{-3} = 707\,\Omega$$

The Quality factor is

$$Q = \frac{\omega_0}{\beta} = 3$$

The cutoff frequencies

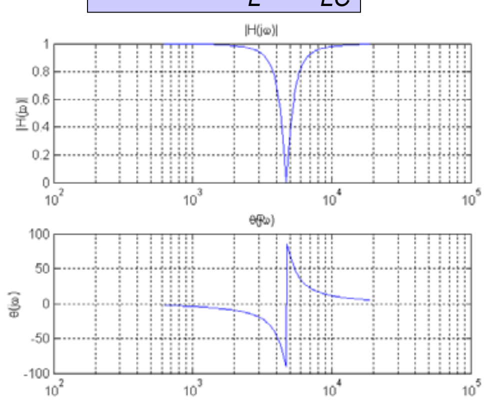
$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + (\omega_0)^2} = 3992 \text{ rad/s}$$

$$\omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + (\omega_0)^2} = 5562.2 \text{ rad/s}$$

Frequency Response of a Circuit

R=707 Ω , L=450 mH, C=100 nF, Plot F=0 – 3 KHz.

$$H(j\omega) = \frac{-\omega^2 + \frac{1}{LC}}{-\omega^2 + \frac{R}{L} j\omega + \frac{1}{LC}}$$



MatLab

```
>> R=707;
>> L=0.450;
>> C=0.0000001;
>> f=100:10:3000;
>> w=2*pi*f;
>> h=abs((-w.^2+1/(L*C))./(-
w.^2+(R/L)*(j*w)+(1/(L*C))));
>> subplot (2,1,1)
>> semilogx(w,h)
>> grid on
>> title('|H(j\omega)|')
>> xlabel ('\omega')
>> ylabel ('|H(j\omega)|')
>> subplot (2,1,2)
>> theta=angle ((-
w.^2+1/(L*C))./(-
w.^2+(R/L)*(j*w)+(1/(L*C))));
>> degree=theta*180/pi;
>> semilogx(w,degree)
>> grid on
>> title('\theta(j\omega)')
>> xlabel('\omega')
>> ylabel('\theta(j\omega)')
```

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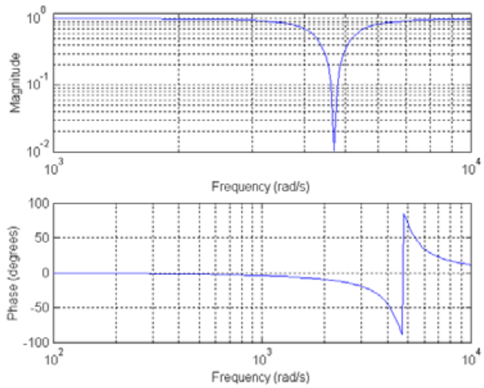
R=707 Ω , L=450 mH, C=100 nF, Plot F=0 – 3 KHz.

$$H(s) = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$



MatLab

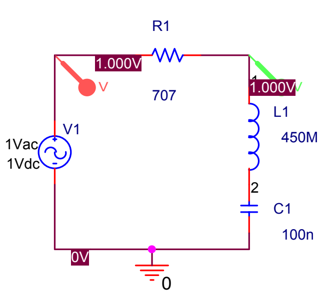
```
>> R=707;
>> L=0.450;
>> C=0.0000001;
>> a=1/(L*C)
>> b=R/L
>> n=[1 0 a];
>> dn=[1 b a];
>> freqs(n,dn)
```

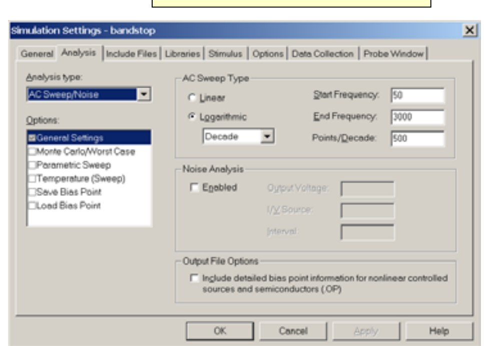

Frequency Response of a Circuit

Example

OrCad Capture

Edit Simulation Profile





Frequency Response of a Circuit

PSpice Result