ECE 307 Network Analysis III

Frequency Response of a Circuit

Band-Pass Filter

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Frequency Response of a Circuit

Band-Pass Filter

Three important parameters

Center frequency (or resonance frequency), ω_0 is defined as the frequency for which a the transfer function of a circuit is purely real

$$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}}$$

Bandwidth, β is the width of the passband

$$\beta = \omega_{c2} - \omega_{c1} \qquad \beta = f_{c2} - f_{c1}$$

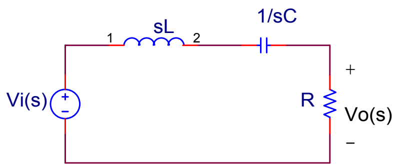
Qualty factor is the ration of the center frequency ω_0 to the bandwidth β.

$$Q = \frac{\omega_0}{\omega_{c2} - \omega_{c1}} \qquad Q = \frac{\omega_0}{\beta} \qquad Q = \frac{f_0}{f_{c2} - f_{c1}}$$

Frequency Response of a Circuit

Band-Pass Filter

A Serial RLC Circuit



$$\frac{V_0(s)}{V_i(s)} = \frac{R}{sL + R + \frac{1}{sC}}$$

$$H(s) = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

To find frequency response, substitute s=jω in equation

$$H(j\omega) = \frac{\frac{R}{L}j\omega}{-\omega^2 + \frac{R}{L}j\omega + \frac{1}{LC}}$$

Magnitude Response

$$|H(j\omega)| = \frac{\frac{R}{L}\omega}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{R}{L}\omega\right)^2}}$$

Phase Response

$$\theta(j\omega) = 90^\circ - \tan^{-1}\left(\frac{\omega\frac{R}{L}}{\frac{1}{LC} - \omega^2}\right)$$

Frequency Response of a Circuit

A Serial RLC Circuit

At resonance frequency, the transfer function will be real.
Or system total impedance will be real

$$j\omega_0 L + \frac{1}{j\omega_0 C} = 0$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

H_{max} will be at $|H(j\omega_0)|$

$$|H(j\omega_0)| = \frac{\frac{R}{L}\omega_0}{\sqrt{\left(\frac{1}{LC} - \omega_0^2\right)^2 + \left(\frac{R}{L}\omega_0\right)^2}}$$

substitute $\omega_0 = \sqrt{1/LC}$

Result

$$|H(j\omega_0)| = \frac{\frac{R}{L}\sqrt{\frac{1}{LC}}}{\sqrt{\left(\frac{1}{LC} - \frac{1}{LC}\right)^2 + \left(\frac{R}{L}\sqrt{\frac{1}{LC}}\right)^2}} = 1$$

Frequency Response of a Circuit

A Serial RLC Circuit

Set $(1/\sqrt{2})H_{max}$ to find cutoff frequencies

$$\frac{1}{\sqrt{2}} = \frac{\frac{R}{L}\omega_c}{\sqrt{\left(\frac{1}{LC}-\omega_c^2\right)^2+\left(\frac{R}{L}\omega_c\right)^2}}$$

$$\frac{1}{2}\left[\left(\frac{1}{LC}-\omega_c^2\right)^2+\left(\frac{R}{L}\omega_c\right)^2\right]=\left(\frac{R}{L}\omega_c\right)^2$$

$$\left(\frac{1}{LC}-\omega_c^2\right)^2 = 2\left(\frac{R}{L}\omega_c\right)^2-\left(\frac{R}{L}\omega_c\right)^2$$

$$\left(\frac{1}{LC}-\omega_c^2\right)=\mp\left(\frac{R}{L}\omega_c\right)$$

Result

$$\omega_c^2+\frac{R}{L}\omega_c-\frac{1}{LC}=0 \Rightarrow \omega_{c1}=-\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\left(\frac{1}{LC}\right)}$$

$$\omega_c^2-\frac{R}{L}\omega_c-\frac{1}{LC}=0 \Rightarrow \omega_{c2}=\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\left(\frac{1}{LC}\right)}$$

Confirm

$$\omega_0=\sqrt{\omega_{c1}\omega_{c2}}=\sqrt{\frac{1}{LC}}$$

Frequency Response of a Circuit

A Serial RLC Circuit

The Bandwidth β is

$$\beta=\omega_{c2}-\omega_{c1}$$
$$=\left[\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\left(\frac{1}{LC}\right)}\right]-\left[-\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\left(\frac{1}{LC}\right)}\right]$$

$$\beta=\frac{R}{L}$$

The Quality factor Q is

$$Q=\frac{\omega_0}{\beta}=\frac{\sqrt{\frac{1}{LC}}}{\frac{R}{L}}$$

$$Q=\sqrt{\frac{L}{CR^2}}$$

$$Q=\frac{f_0}{f_{c2}-f_{c1}}$$

Frequency Response of a Circuit

A Serial RLC Circuit

The cutoff frequencies in terms of β and ω_0

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + (\omega_0)^2}$$

$$\omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + (\omega_0)^2}$$

The cutoff frequencies in terms of Q and ω_0

$$\omega_{c1} = \omega_0\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

$$\omega_{c1} = \omega_0\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

Frequency Response of a Circuit

Example Using serial RLC circuit, design band pass filter that select 1-10KHz frequency band for a graphic equalizer in your amplifier

We can use different approaches. Let's find the center frequency first.

$$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{\frac{1}{LC}}$$

$$f_0 = \sqrt{f_{c1}f_{c2}} = \sqrt{1000 * 10000} = 3162.28\ Hz$$

Choose capacitor value as $1\mu F$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

$$L = \frac{1}{\omega_0^2 C} = \frac{1}{(2\pi 3162.28)^2(10^{-6})} = 2.533\,\text{mH}$$

The Quality factor is

$$Q = \frac{f_0}{f_{c2} - f_{c1}} = \frac{3162.28}{10000 - 1000} = 0.3514$$

The Resistor is

$$Q = \sqrt{\frac{L}{CR^2}}$$

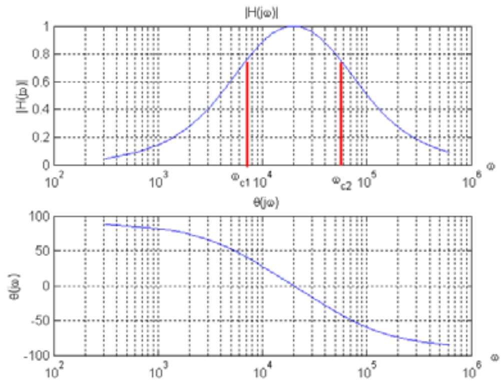
$$R = \sqrt{\frac{L}{CQ^2}} = \sqrt{\frac{0.00253}{10^{-6}(0.3514)^2}} = 143.24\Omega$$

Frequency Response of a Circuit

R=143.25 Ω , L=2.533 mH, C=1 μF, Plot F=50 – 100 KHz.

$$H(j\omega)=\frac{\frac{R}{L}j\omega}{-\omega^2+\frac{R}{L}j\omega+\frac{1}{LC}}$$

```
>> R=143.25;
>> L=0.002533;
>> C=0.000001;
>> f=0:50:100000;
>> w=2*pi*f;
>> h=abs((R/L).*(j*w)./(-w.^2+(R/L)*(j*w)+(1/(L*C))));
>> subplot (2,1,1)
>> semilogx(w,h)
>> grid on
>> title('|H(j\omega)|')
>> xlabel ('\omega')
>> ylabel ('|H(j\omega)|')
>> subplot (2,1,2)
>> theta=angle((R/L).*(j*w)./(-w.^2+(R/L)*(j*w)+(1/(L*C))));
>> degree=theta*180/pi;
>> semilogx(w,degree)
>> grid on
>> title('\theta(j\omega)')
>> xlabel('\omega')
>> ylabel('\theta(j\omega)')
```



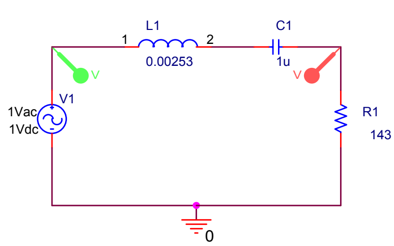
ω_{c1}=6283 rad/s

ω_{c2}=62,831 rad/s

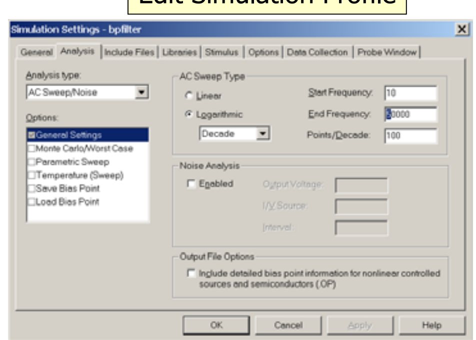
Frequency Response of a Circuit

Example

OrCad Capture



Edit Simulation Profile



Frequency Response of a Circuit

PSpice Result