ECE307

Frequency Response of a Circuit

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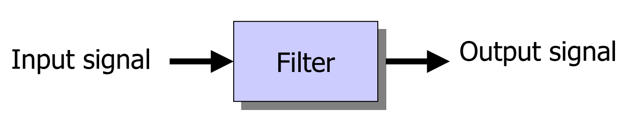
Frequency Response of a Circuit

Some Preliminaries

Analysis of a circuit with varying frequency of a sinusoidal sources is called the **frequency response** of a circuit

Frequency selection in the circuits are called filters because of their ability to filter out certain input signals on the basis of frequency



Frequency Response of a Circuit

Some Preliminaries

We remember that the transfer function is the output voltage to the input voltage of a circuit in s-domain is.

$$H(s) = \frac{V_0(s)}{V_i(s)}$$

Using sinusoidal source, the transfer function will be the magnitude and phase of output voltage to the magnitude and phase of input voltage of a circuit .

In this case we will use ($j\omega$) instead of s .

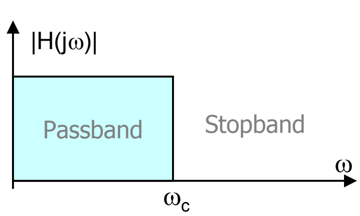
$$H(j\omega) = \frac{V_0(j\omega)}{V_i(j\omega)}$$

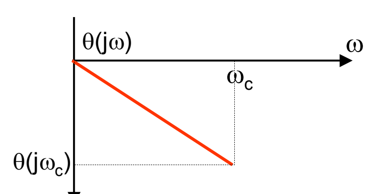
Frequency Response of a Circuit

Frequency Response

Using transfer function of circuit, we plot a frequency response of the circuit for both amplitude and phase with changing source frequency

One graph of $|H(j\omega)|$ versus frequency $j\omega$. It is called the **Magnitude plot.**
One graph of $\theta(j\omega)$ versus frequency ω. It is called the **Phase Angle plot**





ω_c : Cutoff frequency

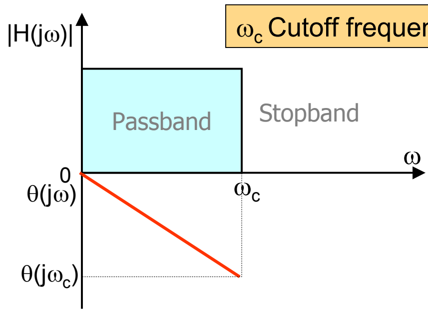
Frequency Response of a Circuit

Filter

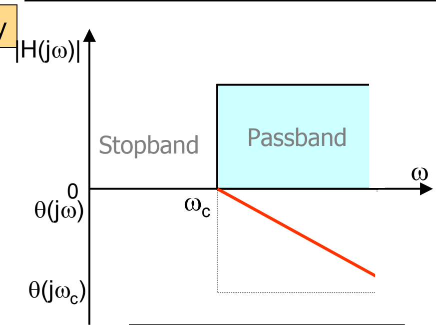
A **Low-Pass filter** passes signals at frequencies lower than the cutoff frequency from the input to the output

A **High-Pass filter** passes signals at frequencies higher than the cutoff frequency from the input to the output

ω_c Cutoff frequency



Ideal Low-Pass Filter



Ideal High-Pass Filter

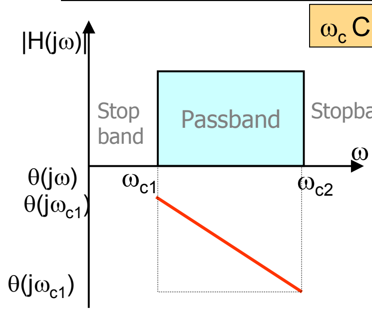
Frequency Response of a Circuit

Filter

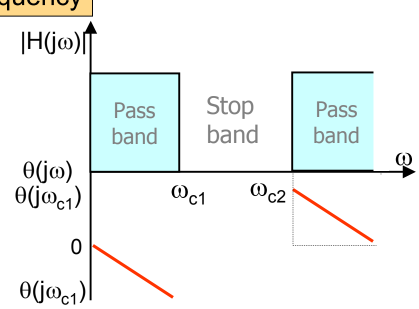
A **Band-Pass filter** passes signals within the band defined by two cutoff frequencies from the input to the output

A **Band-reject filter** passes signals outside the band defined by two cutoff frequencies from the input to the output

ω_c Cutoff frequency



Ideal Band-Pass Filter



Ideal Band-Reject Filter

Frequency Response of a Circuit

Cutoff Frequency

The transfer function magnitude is decreased by the factor $1/\sqrt{2}$ from its maximum value is called **cutoff frequency**

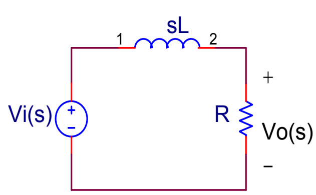
$$|H(j\omega_c)| = \frac{1}{\sqrt{2}}|H_{max}|$$

$|H_{max}|$ is the maximum magnitude of the transfer function

Frequency Response of a Circuit

Low-Pass Filter

A Serial RL Circuit



$$\frac{V_0(s)}{V_i(s)} = \frac{R}{sL + R}$$

$$H(s) = \frac{\frac{R}{L}}{s + \frac{R}{L}}$$

To find frequency response, substitute $s=j\omega$ in equation

$$H(j\omega) = \frac{\frac{R}{L}}{j\omega + \frac{R}{L}}$$

Magnitude Response

$$|H(j\omega)| = \frac{\frac{R}{L}}{\sqrt{\omega^2 + \left(\frac{R}{L}\right)^2}}$$

Phase Response

$$\theta(j\omega) = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

Frequency Response of a Circuit

A Serial RL Circuit

When ω=0

$$|H(j0)| = \frac{\frac{R}{L}}{\sqrt{0^2 + \left(\frac{R}{L}\right)^2}} = 1$$

$$\theta(j0) = -\tan^{-1}\left(\frac{0L}{R}\right) = 0^\circ$$

When ω=∞

$$|H(j\infty)| = \frac{\frac{R}{L}}{\sqrt{\infty^2 + \left(\frac{R}{L}\right)^2}} = 0$$

$$\theta(j\infty) = -\tan^{-1}\left(\frac{\infty L}{R}\right) = -90^\circ$$

To find Cutoff Frequency

$$|H(j\omega_c)| = \frac{\frac{R}{L}}{\sqrt{\omega_c^2 + \left(\frac{R}{L}\right)^2}} = \frac{1}{\sqrt{2}}$$

Result

$$\omega_c = \frac{R}{L}$$

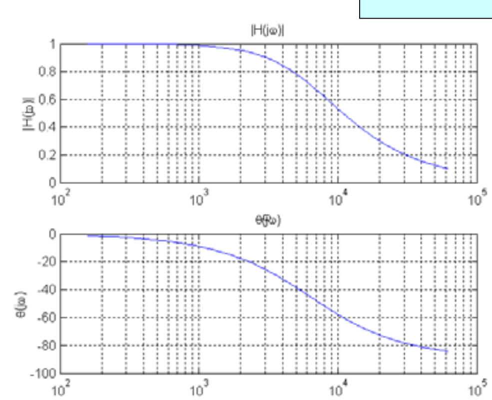
Frequency Response of a Circuit

Example

R=1KΩ F=1KHz. L=? Plot H(jω).

$$L = \frac{R}{\omega_c} = \frac{1000}{2 * \pi * 1000} = 0.159\ H$$

$$H(j\omega) = \frac{\frac{1000}{0.159}}{j\omega + \frac{1000}{0.159}}$$



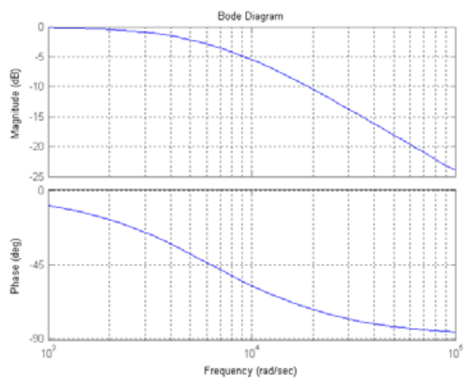

```
>> R=1000;
>> L=0.159;
>> f=0:25:10000;
>> w=2*pi*f;
>> subplot (2,1,1)
>> h=abs((R/L)./(j*w+(R/L)));
>> semilogx(w,h)
>> grid on
>> title('|H(j\omega)|')
>> xlabel ('\omega')
>> ylabel ('|H(j\omega)|')
>> theta=angle((R/L)./(j*w+(R/L)));
>> subplot (2,1,2)
>> degree=theta*180/pi;
>> semilogx(w,degree)
>> grid on
>> title('\theta(j\omega)')
>> xlabel('\omega')
>> ylabel('\theta(j\omega)')
```

Frequency Response of a Circuit

Example

R=1KΩ F=1KHz. L=? Plot H(jω).

$$H(s) = \frac{R/L}{s + R/L}$$

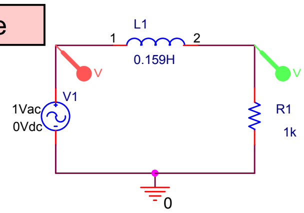


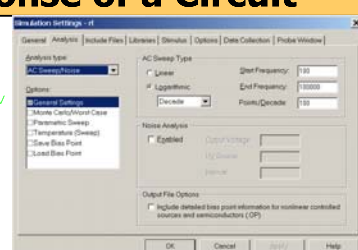
Matlab

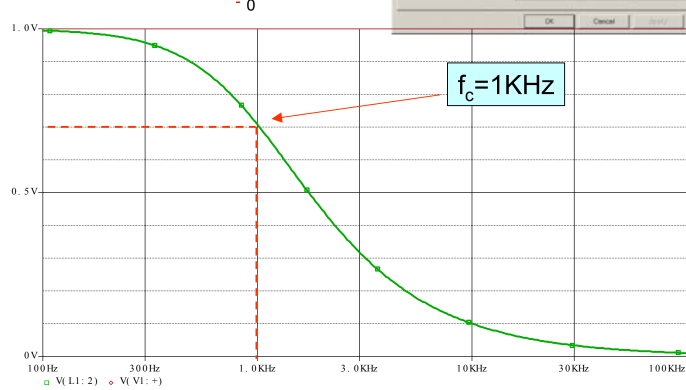
```
>> syms s
>> n=[0 1000/0.159];
>> d=[1 1000/0.159];
>> g=tf(n,d)

Transfer function:
  6289
--------
s + 6289

>> bode (g)
>> grid on
```


Frequency Response of a Circuit

Example



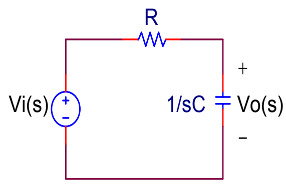




Frequency Response of a Circuit

Low-Pass Filter

A Serial RC Circuit



$$\frac{V_0(s)}{V_i(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}}$$

$$H(s) = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$

To find frequency response, substitute s=jω in equation

$$H(j\omega) = \frac{\frac{1}{RC}}{j\omega + \frac{1}{RC}}$$

Magnitude Response

$$|H(j\omega)| = \frac{\frac{1}{RC}}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}}$$

Phase Response

$$\theta(j\omega) = -\tan^{-1}(\omega RC)$$

Frequency Response of a Circuit

A Serial RC Circuit

When ω=0

$$|H(j0)| = \frac{\frac{1}{RC}}{\sqrt{0^2 + \left(\frac{1}{RC}\right)^2}} = 1$$

$$\theta(j0) = -\tan^{-1}(0RC) = 0^\circ$$

When ω=∞

$$|H(j\infty)| = \frac{\frac{1}{RC}}{\sqrt{\infty^2 + \left(\frac{1}{RC}\right)^2}} = 0$$

$$\theta(j\infty) = -\tan^{-1}(\infty RC) = -90^\circ$$

To find Cutoff Frequency

$$|H(j\omega_c)| = \frac{\frac{1}{RC}}{\sqrt{\omega_c^2 + \left(\frac{1}{RC}\right)^2}} = \frac{1}{\sqrt{2}}$$

Result

$$\omega_c = \frac{1}{RC}$$

Frequency Response of a Circuit

Example A series RC low-pass filter cutoff frequency is 8KHz. R=10KΩ Find the capacitor value

$$\omega_c = \frac{1}{RC} \qquad C = \frac{1}{\omega_c R} \qquad C = \frac{1}{2 * \pi * 8000 x 10000} = 1.99\, nF$$

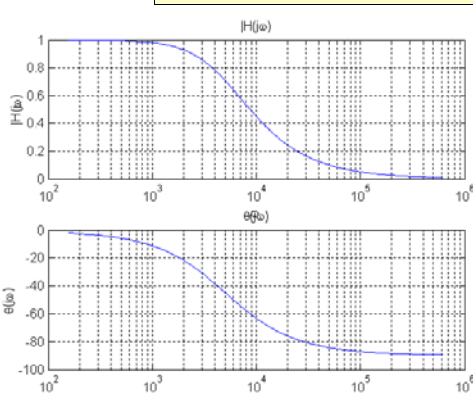
Example A series RL low-pass filter cutoff frequency is 2KHz. R=5KΩ Find the inductor value. Find |H(jω)| at 50 KHz?

$$\omega_c = \frac{R}{L} \qquad L = \frac{R}{\omega_c} \qquad L = \frac{5000}{2000} = 2.5\, H$$

$$\left|H(j\omega)\right|\Big|_{50KHz} = \frac{\frac{R}{L}}{\sqrt{\omega^2 + \left(\frac{R}{L}\right)^2}} = \frac{\frac{5000}{2.5}}{\sqrt{(2 * \pi * 50000)^2 + \left(\frac{5000}{2.5}\right)^2}} = 0.0635$$

Frequency Response of a Circuit

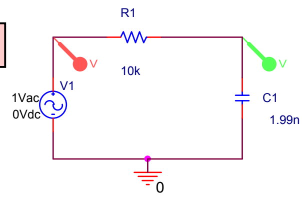
Example A series RC low-pass filter cutoff frequency is 8KHz. R=10KΩ, C=1.99 nF

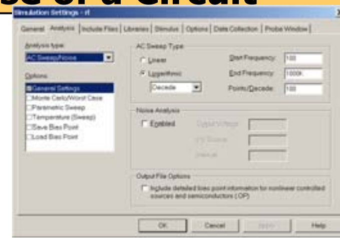


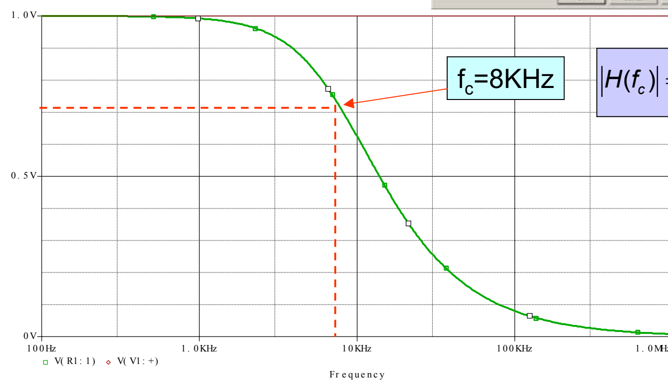

```
r=10000;
c=19.9*10^-9;

f=0:25:100000;
w=2*pi*f;
h=abs((1/(r*c))./(j*w+1/(r*c)));
subplot (2,1,1)
semilogx(w,h)
grid on
title('|H(j\omega)')
xlabel ('\omega')
ylabel ('|H(j\omega)')
theta=angle((1/(r*c))./(j*w+1/(r*c))
);
subplot (2,1,2)
degree=theta*180/pi;
semilogx (w, degree)
grid on
title('\theta(j\omega)')
xlabel('\omega')
ylabel('\theta(j\omega)')
```

Frequency Response of a Circuit

Example



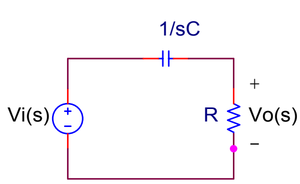




$$|H(f_c)| = \frac{1}{\sqrt{2}}$$

Frequency Response of a Circuit

High-Pass Filter A Serial RC Circuit



$$\frac{V_0(s)}{V_i(s)} = \frac{R}{R + \frac{1}{sC}}$$

$$H(s) = \frac{s}{s + \frac{1}{RC}}$$

To find frequency response, substitute s=jω in equation

$$H(j\omega) = \frac{j\omega}{j\omega + \frac{1}{RC}}$$

Magnitude Response

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}}$$

Phase Response

$$\theta(j\omega) = 90^\circ - \tan^{-1}(\omega RC)$$

Frequency Response of a Circuit

A Serial RC Circuit

When ω=0

$$|H(j0)| = \frac{0}{\sqrt{0^2 + \left(\frac{1}{RC}\right)^2}} = 0$$

$$\theta(j0) = 90^\circ - \tan^{-1}\left(\frac{0}{RC}\right) = 90^\circ$$

When ω=∞

$$|H(j\infty)| = \frac{\infty}{\sqrt{\infty^2 + \left(\frac{1}{RC}\right)^2}} = 1$$

$$\theta(j\infty) = 90^\circ - \tan^{-1}\left(\frac{\infty}{RC}\right) = 0^\circ$$

To find Cutoff Frequency

$$|H(j\omega_c)| = \frac{\omega_c}{\sqrt{\omega_c^{\,2} + \left(\frac{1}{RC}\right)^2}} = \frac{1}{\sqrt{2}}$$

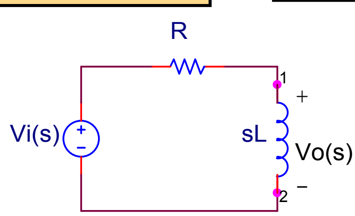
Result

$$\omega_c = \frac{1}{RC}$$

Frequency Response of a Circuit

High-Pass Filter

A Serial RL Circuit



$$\frac{V_0(s)}{V_i(s)} = \frac{sL}{sL + R}$$

$$H(s) = \frac{s}{s + \frac{R}{L}}$$

To find frequency response, substitute s=jω in equation

$$H(j\omega) = \frac{j\omega}{j\omega + \frac{R}{L}}$$

Magnitude Response

$$|H(j\omega)| = \frac{\omega}{\sqrt{\omega^2 + \left(\frac{R}{L}\right)^2}}$$

Phase Response

$$\theta(j\omega) = 90^\circ - \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Frequency Response of a Circuit

A Serial RL Circuit

When ω=0

$$|H(j0)| = \frac{0}{\sqrt{0^2 + \left(\frac{R}{L}\right)^2}} = 0$$

$$\theta(j0) = 90^\circ - \tan^{-1}\left(\frac{0L}{R}\right) = 90^\circ$$

When ω=∞

$$|H(j\infty)| = \frac{\infty}{\sqrt{\infty^2 + \left(\frac{R}{L}\right)^2}} = 1$$

$$\theta(j\infty) = 90^\circ - \tan^{-1}\left(\frac{\infty L}{R}\right) = 0^\circ$$

To find Cutoff Frequency

$$|H(j\omega_c)| = \frac{\omega_c}{\sqrt{\omega_c^{\ 2} + \left(\frac{R}{L}\right)^2}} = \frac{1}{\sqrt{2}}$$

Result

$$\omega_c = \frac{R}{L}$$

Frequency Response of a Circuit

Example

Define R and L values for a high pass filter with a cutoff frequency of 10KHz. Find |H(jω)|at 5 KHz

$$\omega_c = \frac{R}{L}$$

We can't calculate R and L values independently. We can select R or L values then define the other

Let $R = 1K\Omega$

$$L = \frac{R}{\omega_c}$$

Result

$$L = \frac{1000}{2 * \pi * 10000} = 15.9\,mH$$

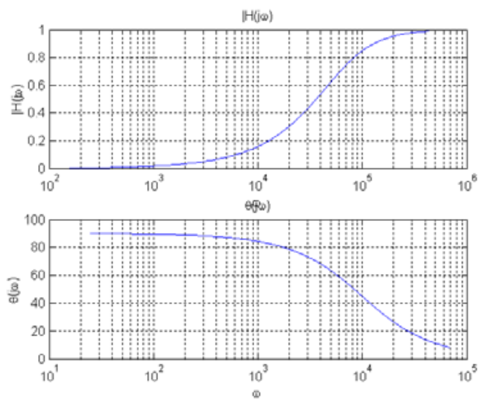
$$|H(j\omega)|\big|_{5KHz} = \frac{\omega}{\sqrt{\omega^2 + \left(\frac{R}{L}\right)^2}} = \frac{2 * \pi * 5000}{\sqrt{(2 * \pi * 5000)^2 + \left(\frac{1000}{0.0159}\right)^2}} = 0.4469$$

Frequency Response of a Circuit

Example

A RL high pass filter with a cutoff frequency of 10KHz. $R = 1K\Omega$

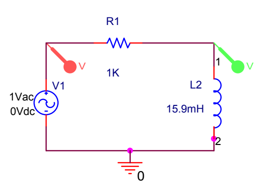
$$L = \frac{1000}{2 * \pi * 10000} = 15.9\,mH$$

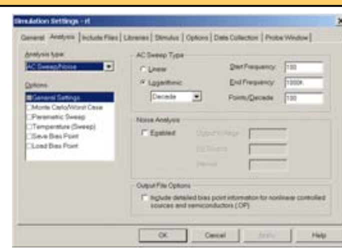


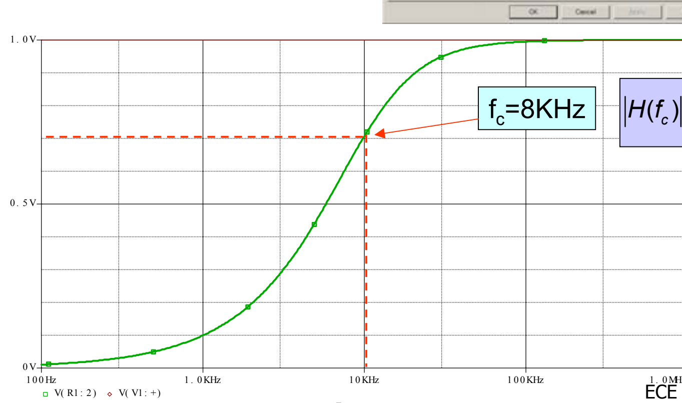

```
R=1000;
L=15.9*10^-3;

f=0:25:70000;
w=2*pi*f;
h=abs((j*w)./(j*w+R/L));
subplot (2,1,1)
semilogx(w,h)
grid on
title('|H(j\omega)')
xlabel ('\omega')
ylabel ('|H(j\omega)')
theta=angle((j*w)./(j*w+R/L));
subplot (2,1,2)
plot (w, theta)
degree=theta*180/pi;
semilogx (f, degree)
grid on
title('\theta(j\omega)')
xlabel('\omega')
ylabel('\theta(j\omega)')
```


Frequency Response of a Circuit

Example







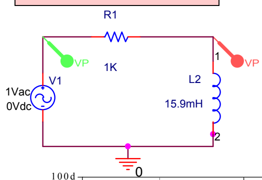
f_c=8KHz

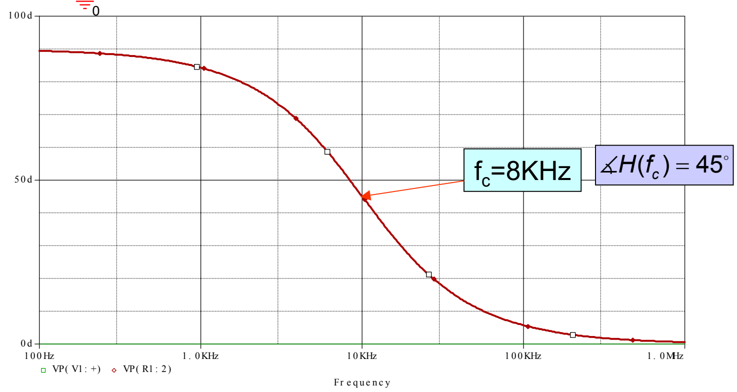
$$|H(f_c)| = \frac{1}{\sqrt{2}}$$

Frequency Response of a Circuit

Plotting phase:

Take the probe Phase of Voltage, which is under Pspice, Markers, and Advanced.
Marked the node you want to see phase

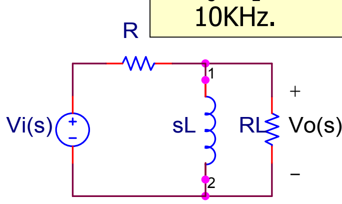




Frequency Response of a Circuit

Example

Let's place load resister in parallel to inductor in RL high-pass filter shown in the figure
a. Find the transfer function
b. $R_s=R_L=1K\Omega$, find L value for cutoff frequency at 10KHz.



$$\frac{V_0(s)}{V_i(s)} = \frac{\dfrac{R_L sL}{R_L + sL}}{R + \dfrac{R_L sL}{R_L + sL}}$$

$$H(s) = \frac{\dfrac{R_L}{R+R_L} s}{s + \dfrac{R_L}{R+R_L}\dfrac{R}{L}} = \frac{Ks}{s + K\dfrac{R}{L}}$$

where $K = \dfrac{R_L}{R+R_L}$ $\quad K = \dfrac{1}{1+1} = 0.5$

$$\omega_c = K\frac{R}{L}$$

Result

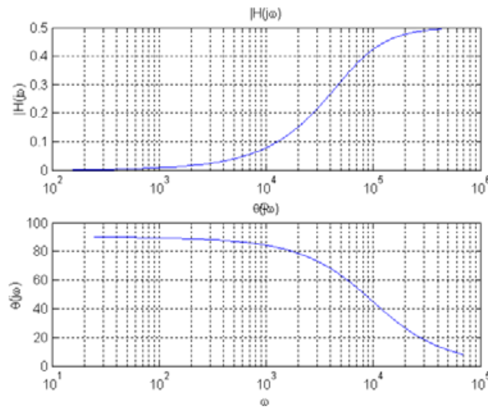
$$L = K\frac{R}{\omega_c} = 0.5\frac{1}{2 * \pi * 10} = 7.95\,mH$$

Frequency Response of a Circuit

Example

$R_s=R_L=1K\Omega$, L=7.95 mH High-pass filter cutoff frequency at 10KHz.

$$H(j\omega)=\frac{j\omega K}{j\omega+K\frac{R}{L}}$$




```
R=1000;
RL=1000;
L=7.95*10^-3;
K=RL/(R+RL)

f=0:25:70000;
w=2*pi*f;
h=abs((j*w*K)./(j*w+K*R/L));
subplot (2,1,1)
semilogx(w,h)
grid on
title('|H(j\omega)')
xlabel ('\omega')
ylabel ('|H(j\omega)')
theta=angle((j*w*K)./(j*w+K*R/L));
subplot (2,1,2)
plot (w, theta)
degree=theta*180/pi;
semilogx (f, degree)
grid on
title('\theta(j\omega)')
xlabel('\omega')
ylabel('\theta(j\omega)')
grid on
```