ECE 307

Fourier Series-II

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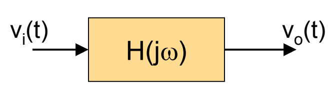
Fourier Series

Calculation of Steady State Responses to Periodic Inputs



Let's have sinusoidal input signal as

$$v_i(t) = V_m Cos(\omega_0 t + \theta)$$

If we know the system transfer function as $H(j\omega)$, then we can find the output as

$$v_0(t) = V_m \left|H(j\omega_0)\right| Cos[\omega_0 t + \theta \angle H(j\omega_0)]$$

t

Fourier Series

Calculation of Steady State Responses to Periodic Inputs

We know that a periodic signal vi(t) can be expressed in a Fourier Series expansion

$$v_i(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$$

We can also represent the output signal as

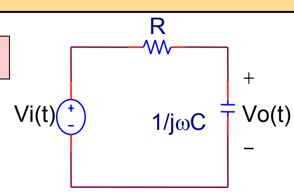
$$v_0(t) = c_0 |H(j0)| + \sum_{n=1}^{\infty} c_n |H(jn\omega_0)| \cos[n\omega_0 t + \theta_n + \angle H(jn\omega_0)]$$

Which is same sum of harmonic as at the input but the magnitudes and phases are changed

$C_n |H(jn\omega_0)|$ $\theta_n + \angle H(jn\omega_0)$

Fourier Series

Example:



$$H(j\omega) = \frac{\frac{1}{RC}}{j\omega + \frac{1}{RC}}$$

Magnitude Response

$$|H(j\omega)| = \frac{\frac{1}{RC}}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}}$$

Phase Response

$$\theta(j\omega) = -\tan^{-1}(\omega RC)$$

A series RC low-pass filter cutoff frequency is 8KHz. R=10KΩ, C=1.99 nF

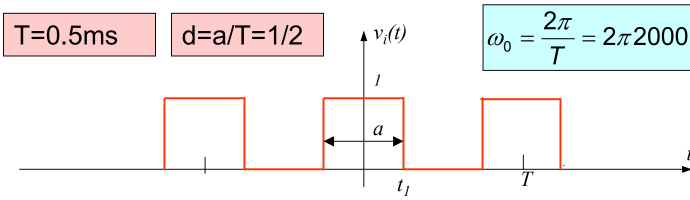
$$|H(j\omega)| = \frac{(4.26)10^4}{\sqrt{\omega^2 + ((4.26)10^4)^2}}$$

Fourier Series

T=0.5ms

d=a/T=1/2

$\omega_0 = \frac{2\pi}{T} = 2\pi 2000$ rad/s



$$v_i(t) = \frac{1}{2} + \frac{2}{\pi}\cos(\omega_0 t) - \frac{2}{3\pi}\cos(3\omega_0 t) + \frac{2}{5\pi}\sin(5\omega_0 t) - \frac{2}{7\pi}\cos(7\omega_0 t)...$$

$|H(j0)| = 1$

$$|H(j\omega_0)| = \frac{(4.26)10^4}{\sqrt{\omega_0^2 + \left((4.26)10^4\right)^2}} = 0.97$$

$$\theta(j\omega_0) = -\tan^{-1}(\omega_0 RC) = -14^\circ$$

$|H(j3\omega_0)| = 0.80$ $\theta(j3\omega_0) = -36.87^\circ$

$|H(j5\omega_0)| = 0.6246$ $\theta(j5\omega_0) = -51.35^\circ$

$|H(j7\omega_0)| = 0.496$ $\theta(j7\omega_0) = -60.26^\circ$

```
>> R=10000;
>> C=1.99e-9;
>> K=1/(R*C);
>> n=1:2:7;
>> wo=2*pi*2000;
>> H=K./((j*n*wo)+K);
>> Habs=abs(H)
>> degree=angle(H)*180/pi
```

Fourier Series

The output voltage

$$v_0(t) = \frac{1}{2}(1) + \frac{2}{\pi}0.97\cos(\omega_0 t - 14^\circ) - \frac{2}{3\pi}0.80\cos(3\omega_0 t - 36.87^\circ) + \frac{2}{5\pi}0.62\sin(5\omega_0 t - 51^\circ) - \frac{2}{7\pi}0.49\cos(7\omega_0 t - 60^\circ)...$$

$$v_0(t) = \frac{1}{2}(1) + \frac{1.94}{\pi}\cos(\omega_0 t - 14^\circ) - \frac{1.6}{3\pi}\cos(3\omega_0 t - 36.87^\circ) + \frac{1.24}{5\pi}\sin(5\omega_0 t - 51^\circ) - \frac{0.98}{7\pi}\cos(7\omega_0 t - 60^\circ)...$$

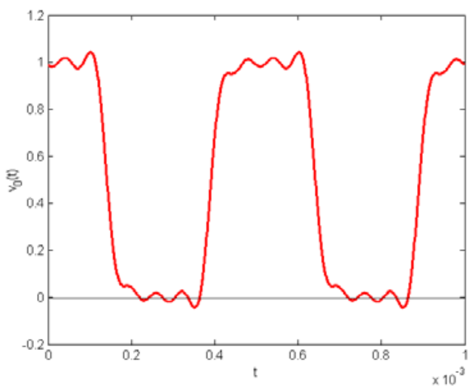
$C_n|H(jn\omega_0)|$

```
>> cn=2./(n*pi).*Habs

cn = 0.6176    0.1697    0.0795    0.0451
```

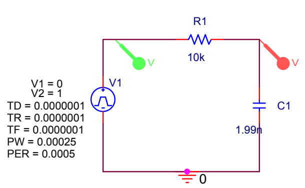
Fourier Series

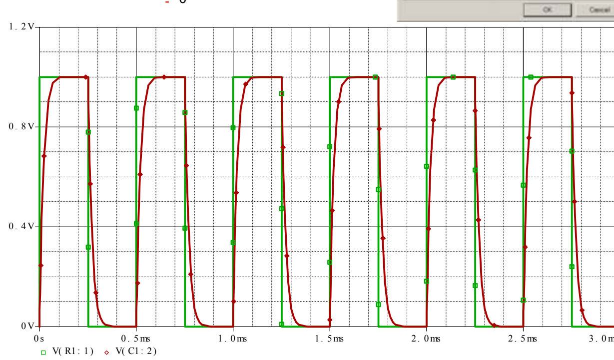
Example:




```
>> R=10000;
>> C=1.99e-9;
>> K=1/(R*C);
>> n=1:2:7;
>> wo=2*pi*2000;
>> H=K./((j*n*wo)+K);
>> Habs=abs(H)
>> degree=angle(H)*180/pi
>> cn=2./(n*pi).*Habs
cn =     0.6176     0.1697
         0.0795     0.0451
>> t=0:0.000001:0.001;
>> c1=0.6176*cos(2*pi*2000*t-
14*pi/180);
>>
c3=0.1697*cos(2*pi*3*2000*t-
36.87*pi/180);
>>
c5=0.0795*cos(2*pi*5*2000*t-
51.35*pi/180);
>>
c7=0.0451*cos(2*pi*7*2000*t-
60.26*pi/180);
>> c=0.5+c1-c3+c5-c7;
>> plot (t,c)
>> xlabel('t')
>> ylabel('v_0(t)')
```

Fourier Series





Fourier Series

Calculation of Steady State Responses to Periodic Inputs expressed as sum of Complex Exponential

$$x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$$

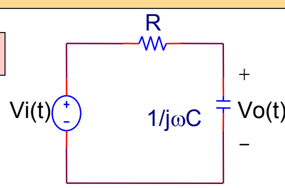
We can also represent the output signal as

$$v_0(t) = \sum_{n=-\infty}^{\infty} X_n H(jn\omega_0) e^{jn\omega_0 t}$$

Remember that $H(-j\omega) = H^*(j\omega)$

Fourier Series

Example:



Transfer function

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

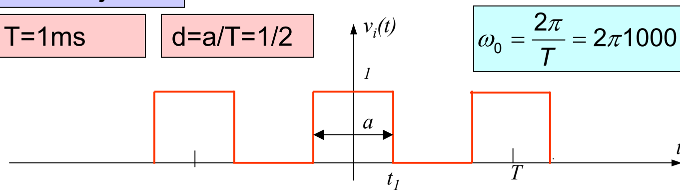
A series RC low-pass filter cutoff frequency is R=1KΩ, C=1 μF

$$H(j\omega) = \frac{1}{1 + j\omega 10^{-3}}$$

T=1ms

d=a/T=1/2

$$\omega_0 = \frac{2\pi}{T} = 2\pi 1000 \text{ rad/s}$$



$$v_i(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} \frac{1}{2} \text{sinc}(\frac{n}{2}) e^{jn\omega_0 t}$$

Fourier Series

The output is

$$v_0(t) = \sum_{n=-\infty}^{\infty} X_n H(jn\omega_0) e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} \frac{\frac{1}{2}\text{sinc}(\frac{n}{2})}{1 + jn\omega_0 10^{-3}} e^{jn\omega_0 t}$$

$$v_0(t) = \sum_{n=-\infty}^{\infty} \frac{\frac{1}{2}\text{sinc}(\frac{n}{2})}{1 + jn2\pi} e^{jn\omega_0 t}$$

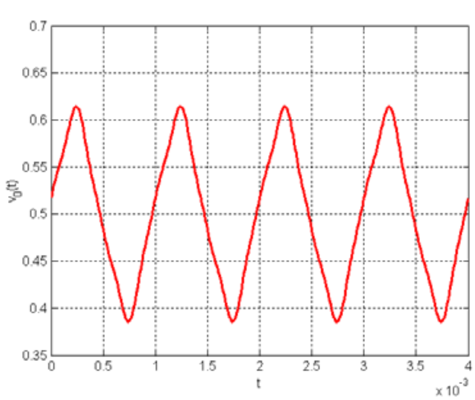
$$v_0(t) = \frac{1}{2} + \frac{0.3183}{1+j2\pi} e^{j\omega_0 t} + \frac{0.3183}{1-j2\pi} e^{-j\omega_0 t} - \frac{0.1061}{1+j6\pi} e^{j3\omega_0 t} - \frac{0.3183}{1-j6\pi} e^{-j3\omega_0 t} + \frac{0.0637}{1+j10\pi} e^{j5\omega_0 t} - \frac{0.0637}{1-j10\pi} e^{-j5\omega_0 t}$$

$$v_0(t) = \frac{1}{2} + 0.05e^{-j81^\circ} e^{j\omega_0 t} + 0.05e^{+j81^\circ} e^{-j\omega_0 t} - 0.0056e^{-j93^\circ} e^{j3\omega_0 t} - 0.0056e^{+j93^\circ} e^{-j3\omega_0 t} + 0.002e^{j88^\circ} e^{j5\omega_0 t} - 0.002e^{-j88^\circ} e^{-j5\omega_0 t}$$

$$v_0(t) = \frac{1}{2} + 0.1\cos(\omega_0 t - 81^\circ) - 0.0112\cos(3\omega_0 t - 93^\circ) + 0.004\cos(5\omega_0 t - 88^\circ)$$

Fourier Series

MatLab




```
>> t=0:0.00001:0.004;
>> wo=2*pi*1000;
>> y1=0.1*cos(wo*t-81*pi/180);
>> y3=0.0112*cos(3*wo*t-
93*pi/180);
>> y5=0.004*cos(5*wo*t-
88*pi/180);
>> y=0.5+y1-y3+y5;
>> plot (t,y)
>> grid on
>> xlabel('t')
>> ylabel('v_0(t)')
```

Fourier Series

PSpice