ECE 307-3

Circuit Analysis in s-Domain

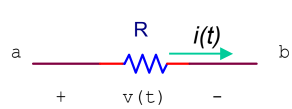
Electrical and Computer Engineering Department
Cal Poly Pomona

Circuit Elements in the s-Domain

The Laplace Transform of V(t) and I(t) are

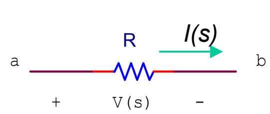
$V(s) = L\{v(t)\}$ $I(s) = L\{i(t)\}$

A Resistor in the s Domain



$v(t) = Ri(t)$

The Laplace Transform is

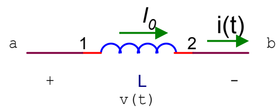


$V(s) = R\, I(s)$

$I(s) = \frac{V(s)}{R}$

Circuit Elements in the s-Domain

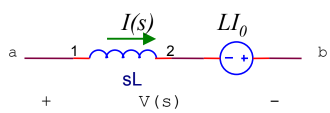
An Inductor in the s Domain



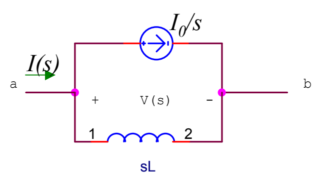
$$v(t) = L\frac{di(t)}{dt}$$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt$$

The Laplace Transform is



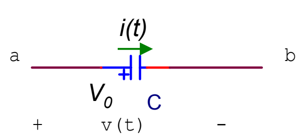
$$V(s) = L\left[sI(s) - i(0^-)\right] = LsI(s) - LI_0$$



$$I(s) = \frac{V(s) + LI_0}{sL} = \frac{1}{sL}V(s) + \frac{I_0}{s}$$

Circuit Elements in the s-Domain

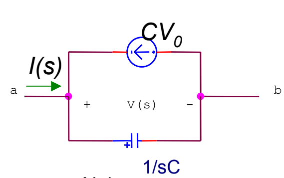
An capacitor in the s Domain



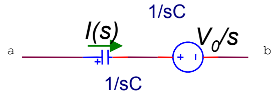
$$i(t) = C\frac{dv(t)}{dt}$$

$$v(t) = \frac{1}{c}\int_{t_0}^{t} i(t)\,dt$$

The Laplace Transform is



$$I(s) = C\left[sV(s) - v(0^-)\right] = CsV(s) - CV_0$$

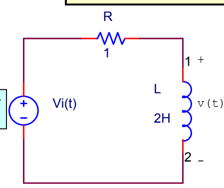


$$V(s) = \frac{I(s) + CV_0}{sC} = \frac{1}{sC}I(s) + \frac{V_0}{s}$$

Circuit Elements in the s-Domain

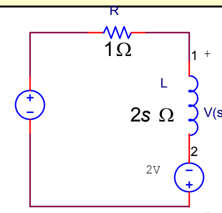
Example: Calculate the voltage v(t) in the following circuit. The initial current I_0=1A

$3e^{-8t}u(t)\ V$



The equivalent s-domain circuit

$\frac{3}{s+8}V$



$$I(s)=\frac{\frac{3}{S+8}+2}{2s+1}=\frac{s+9.5}{(s+8)(s+0.5)}$$

$$V(s)=2sI(s)-2$$

$$V(s)=2s\frac{s+9.5}{(s+8)(s+0.5)}-2$$

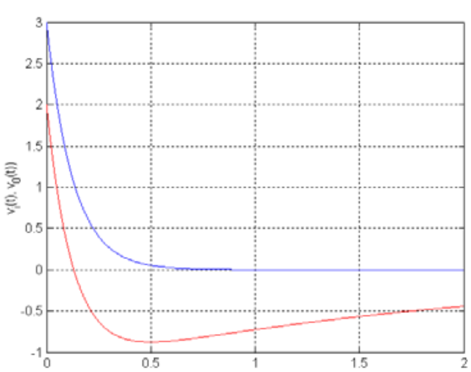
$$V(s)=\frac{2s-8}{(s+8)(s+0.5)}$$

$$V(s)=\frac{3.2}{(s+8)}-\frac{1.2}{(s+0.5)}$$

The result

$$v(t)=\left[3.2e^{-8t}-1.2e^{-0.5t}\right]u(t)\ V$$

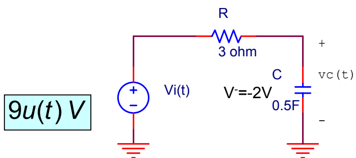
Circuit Elements in the s-Domain



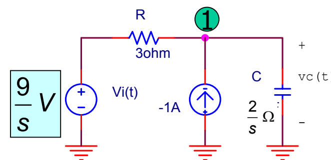

```
>> t=0:0.0001:2;
>> v=3.*exp(-8*t);
>> plot (t,v)
>> hold on
>> vo=3.2*exp(-8*t)-1.2*exp(-
0.5*t);
>> plot (t,vo,'r')
>> grid on
>> xlabel ('t');
>> ylabel('v_i(t), v_0(t))')
```

Circuit Elements in the s-Domain

Example: Calculate the voltage $v_c(t)$ in the following circuit



The equivalent s-domain circuit



Write a single node (1) equation

$$-1=\frac{V_c(s)}{2/s}+\frac{Vc(s)-9/s}{3}$$

Solve for $V_c(s)$

$$V_c(s)=\frac{\frac{18}{s}-6}{3s+2}=\frac{6-2s}{s(s+\frac{2}{3})}$$

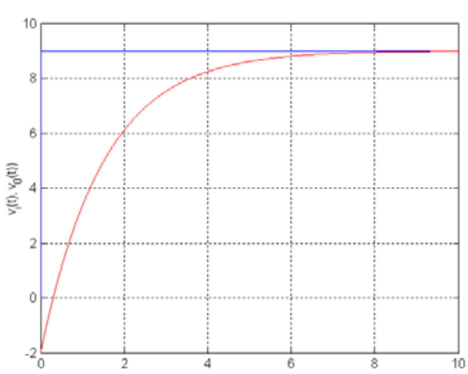
Partial Fraction expansion

$$V_c(s)=\frac{9}{s}-\frac{11}{(s+\frac{2}{3})}$$

The result

$$v(t)=\left[9-11e^{-\frac{2}{3}t}\right]u(t)\ V$$

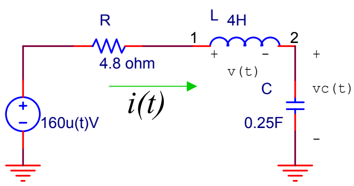
Circuit Elements in the s-Domain




```
>> t=0:0.001:10;
>> v0=9-11*exp(-2/3*t);
>> plot (t,v0)
>> v=9*[t>0];
>> plot (t,v)
>> hold on
>> plot (t,v0, 'r')
>> xlabel ('t');
>> ylabel('v_i(t), v_0(t))')
>> grid on
```

Circuit Elements in the s-Domain

Example: Find I(t) , Find v(t) in the following circuit



Writing current ins-domain

$$I(s)=\frac{\frac{160}{s}}{4.8+sL+\frac{1}{sC}}=\frac{160}{4s^2+4.8s+4}$$

$$I(s)=\frac{40}{s^2+1.2s+1}$$

Partial Fraction expansion

$$I(s)=\frac{40}{s^2+1.2s+1}=\frac{-j25}{s+0.6-j0.8}+\frac{j25}{s+0.6+j0.8}$$

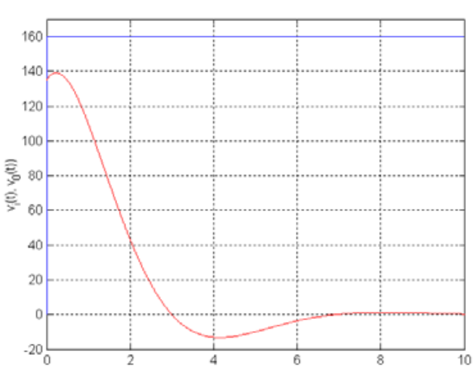
The result $i(t)=\left[50e^{-0.6t}\sin(0.8t)\right]u(t)\ A$

Voltage in inductor

$$V(s)=\frac{160s}{(s^2+1.2s+1)}$$

$$v(t)=\left[200e^{-0.6t}\cos(0.8t+36.87^{\circ})\right]u(t)\ V$$

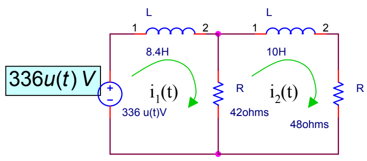
Circuit Elements in the s-Domain



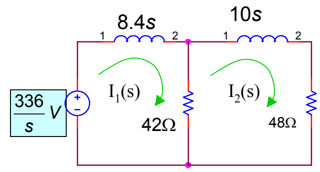

```
>> v0=200*exp(-
0.6*t).*cos(0.8*t+36.87);
>> plot (t,v0)
>> v=160*[t>0];
>> plot (t,v)
>> hold on;
>> v0=200*exp(-
0.6*t).*cos(0.8*t+36.87);
>> plot (t,v0, 'r')
>> xlabel ('t');
>> ylabel('v_i(t), v_0(t))')
>> grid on
>>
```

Circuit Elements in the s-Domain

Example: Calculate the voltage $v_c(t)$ in the following circuit



The equivalent s-domain circuit



The two mesh-current equations

$$\frac{336}{s} = (8.4s + 42)I_1(s) - 42I_2(s)$$

$$0 = -42I_1(s) + (10s + 90)I_2(s)$$

$$\begin{bmatrix} 8.4s + 42 & -42 \\ -42 & 10s + 90 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} 336/s \\ 0 \end{bmatrix}$$

Circuit Elements in the s-Domain

Example: (cont)

$$I_1(s) = \frac{\begin{vmatrix} 336/s & -42 \\ 0 & 10s + 90 \end{vmatrix}}{\begin{vmatrix} 8.4s + 42 & -42 \\ -42 & 10s + 90 \end{vmatrix}}$$

$$I_1(s) = \frac{40(s+9)}{s(s+2)(s+12)}$$

Therefore f(t) can be obtain

$$i_1(t) = \left[15 - 14e^{-2t} - e^{-12t}\right]u(t)\ A$$

$$I_2(s) = \frac{\begin{vmatrix} 8.4s + 42 & 336/s \\ -42 & 0 \end{vmatrix}}{\begin{vmatrix} 8.4s + 42 & -42 \\ -42 & 10s + 90 \end{vmatrix}}$$

$$I_1(s) = \frac{168}{s(s+2)(s+12)}$$

The inverse transform of

$$i_2(t) = \left[7 - 8.4e^{-2t} + 1.4e^{-12t}\right]u(t)\ A$$

Circuit Elements in the s-Domain

MatLab

To find partial fraction

$$X(s) = \frac{10s^2 + 25s}{(18s^3 + 21s^2 + 54s + 28)}$$

```
>> num = [10 25 0];
>> den= [18 21 54 28];
>> [r p y] = residue(num, den)
```


Circuit Elements in the s-Domain

MatLab

To find partial fraction

$$X(s) = \frac{5s^4}{(s^2 + 9)(90s^3 + 18s^2 + 40s + 4)}$$

```
>> d1 = 's^2+9';
>> d2 = '90*s^3+18*s^2+40*s+4';
>> d = symmul(d1,d2);
>> denominator = expand(d);
>> den=sym2poly(denominator);
>> num = [5 0 0 0 0]
>> [r p y] = residue(num, den)
```