ECE307-9

Bode Diagram-2

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Bode Diagram

Dode Diagrams for Complex poles and zeros

Bode diagram consists of two separate plots
The amplitude of H(jω) varies with frequency
The phase angle of H(jω) varies with frequency

Transfer function H(s) contains complex poles

$$H(s) = \frac{K}{s^2 + as + b}$$

The conjugate pair into a single quadratic term

$$H(s) = \frac{K}{(s+\alpha - j\beta)(s+\alpha + j\beta)}$$

Writing quadratic term in more convenient form

$$(s+\alpha)^2 + \beta^2 = s^2 + 2\alpha s + \alpha^2 + \beta^2 = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\omega_n^2 = \alpha^2 + \beta^2$$

ω_n is the corner frequency of quadratic term

$$\zeta\omega_n = \alpha$$

ζ is the damping coefficient of quadratic term

Bode Diagram

Bode Diagrams for Complex poles and zeros

ζ<1 the roots are complex. So assume that ζ<1

H(s) is given

$$H(s)=\frac{K}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$H(s)=\frac{K}{\omega_n^2}\frac{1}{1+\left(\frac{s}{\omega_n}\right)^2+2\zeta\frac{s}{\omega_n}}$$

Replace s with jω

$$H(j\omega)=\frac{K_0}{1-\left(\frac{\omega}{\omega_n}\right)^2+j2\zeta\frac{\omega}{\omega_n}}$$

$$K_0=\frac{K}{\omega_n^2}$$

In polar form

$$H(j\omega)=\frac{K_0}{\left|1-\left(\frac{\omega}{\omega_n}\right)^2+j2\zeta\frac{\omega}{\omega_n}\right|\angle\beta_1}$$

$$\beta_1=\tan^{-1}\left(\frac{2\zeta\frac{\omega}{\omega_n}}{1-\frac{\omega}{\omega_n}}\right)$$

$$A_{dB}=20\log_{10}K_0-20\log_{10}\left|1-\left(\frac{\omega}{\omega_n}\right)^2+j2\zeta\frac{\omega}{\omega_n}\right|$$

$$\theta(\omega)=-\beta_1$$

Bode Diagram

Amplitude Plots

The amplitude of value of $-20\log_{10}\left|1-\left(\frac{\omega}{\omega_n}\right)^2+j2\zeta\frac{\omega}{\omega_n}\right|$

$$-20\log_{10}\left|1-\left(\frac{\omega}{\omega_n}\right)^2+j2\zeta\frac{\omega}{\omega_n}\right|\to 0\quad\text{when } \omega\to 0$$

$$-20\log_{10}\left|1-\left(\frac{\omega}{\omega_n}\right)^2+j2\zeta\frac{\omega}{\omega_n}\right|\to -40\log_{10}\frac{\omega}{\omega_n}\quad\text{when } \omega\to\infty$$

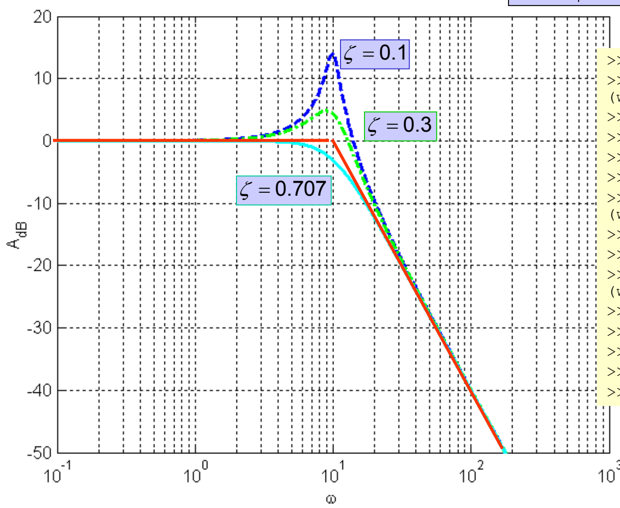
Results

The approximate amplitude plot consists of two straight lines
1. Straight line lies along the 0 dB when $\omega < \omega_n$
2. Straight Line has a slope –40dB/decade when $\omega < \omega_n$

Bode Diagram

Straight-Line Amplitude Plots

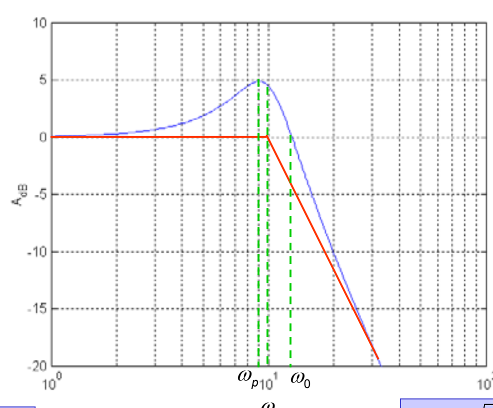
$$-20\log\left|1-\left(\frac{\omega}{\omega_n}\right)^2+j2\zeta\frac{\omega}{\omega_n}\right|$$




```
>> w=0.1:0.01:1000;
>> x=1./((1-
(w/10).^2)+j*2*0.1*(w/10));
>> a=20*log10(abs(x));
>> semilogx(w,a)
>> grid on
>> hold on
>> x=1./((1-
(w/10).^2)+j*2*0.3*(w/10));
>> a=20*log10(abs(x));
>> semilogx(w,a, 'g')
>> x=1./((1-
(w/10).^2)+j*2*0.707*(w/10));
>> a=20*log10(abs(x));
>> semilogx(w,a,'c')
>> xlabel ('\omega')
>> ylabel ('theta(\omega)')
>> ylabel ('\theta(\omega)')
```


Bode Diagram

Straight-Line Amplitude Plots



$$\omega_p = \omega_n\sqrt{1-2\zeta^2}$$

$$\omega_0 = \omega_n\sqrt{2(1-2\zeta^2)} = \sqrt{2}\,\omega_p$$

$$A_{dB}(\omega_p) = -10\log_{10}\left[4\zeta^2(1-2\zeta^2)\right]$$

$$A_{dB}(\omega_n) = -20\log_{10}2\zeta$$

Bode Diagram

Straight-Line Phase Angle Plots

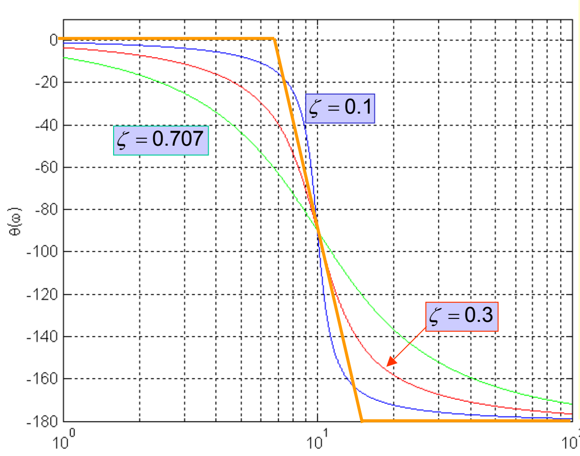
$$H(j\omega) = \frac{K_0}{\left|1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\zeta\frac{\omega}{\omega_n}\right| \angle \beta_1}$$

$$\theta(\omega) = -\beta_1 = -\tan^{-1}\left(\frac{2\zeta\frac{\omega}{\omega_n}}{1 - \frac{\omega}{\omega_n}}\right)$$

The phase angle of Complex poles and zeros has three straight lines approximation

1. The phase angle zero at zero frequency $\omega \rightarrow 0$
2. - 90 degree at the corner frequency ω_n
3. -180 degree close the large frequency $\omega \rightarrow \infty$

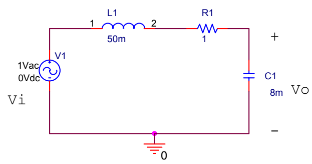
Straight-Line Phase Angle Plots




```
>> w=1:0.01:100;
>> x=1./((1-(w/10).^2)+j*2*0.1*(w/10));
>> degree=180*angle(x)/pi;
>> semilogx(w,degree)
>> grid on
>> hold on
>> x=1./((1-(w/10).^2)+j*2*0.3*(w/10));
>> degree=180*angle(x)/pi;
>> semilogx(w,degree, 'r')
>> x=1./((1-(w/10).^2)+j*2*0.707*(w/10));
>> degree=180*angle(x)/pi;
>> semilogx(w,degree, 'g')
>> xlabel ('\omega')
>> ylabel ('theta(\omega)')
>> ylabel ('\theta(\omega)')
```

Bode Diagram

Example



Find K_o
Bode Plot
Damping coefficient
Amplitude value of ω_p, ω_0, and ω_n

$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{2500}{s^2 + 20s + 2500}$$

$$\omega_n^2 = 2500$$

$$\zeta = \frac{\alpha}{2\omega_n} = \frac{20}{100} = 0.20$$

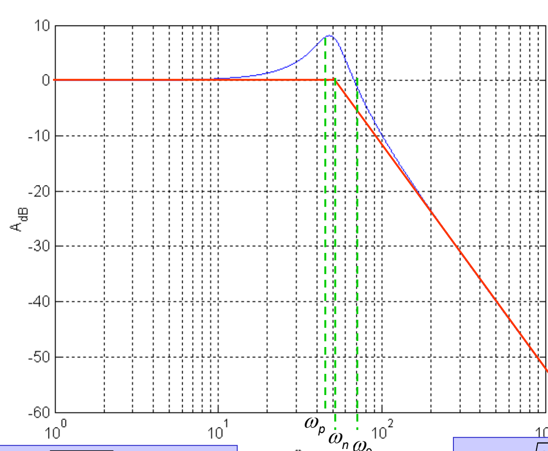
$$H(j\omega) = \frac{K_0}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\zeta\frac{\omega}{\omega_n}}$$

$$H(j\omega) = \frac{1}{1 - \left(\frac{\omega}{50}\right)^2 + j0.4\frac{\omega}{50}}$$

$$K_0 = 1$$

$$A_{dB} = -20\log\left|1 - \left(\frac{\omega}{50}\right)^2 + j0.4\frac{\omega}{50}\right|$$

MatLab




```
>> w=1:0.1:1000;
>> a=-20*log10(abs(1-(w/50).^2+j*0.4*(w/50)));
>> semilogx(w,a)
>> grid on
>> xlabel ('\omega')
>> ylabel ('A_{dB}')
```

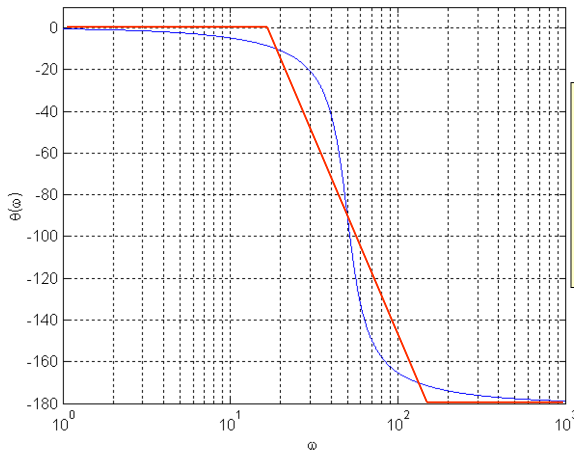
$$\omega_p = \omega_p\sqrt{1-2\zeta^2} = 47.9 \text{ rad/s}$$

$$\omega_0 = \omega_n\sqrt{2(1-2\zeta^2)} = \sqrt{2}\omega_p = 67.82 \text{ rad/s}$$

$$A_{dB}(\omega_p) = -10\log_{10}\left[4\zeta^2(1-2\zeta^2)\right] = 2.2 \text{ dB}$$

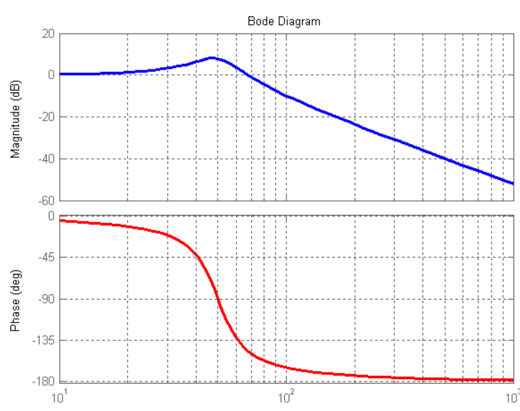
$$A_{dB}(\omega_n) = -20\log_{10}2\zeta = 7.96 \text{ dB}$$

Bode Diagram



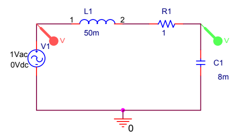

```
>> w=1:0.1:1000;
>> theta=angle(1-
(w/50).^2+j*0.4*(w/50));
>> degree=-theta*180/pi;
>> semilogx(w,degree)
>> grid on
>> xlabel ('\omega')
>>  ylabel
('\theta(\omega)')
```

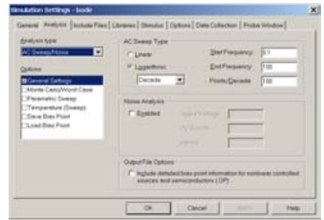

Bode Plot in MatLab

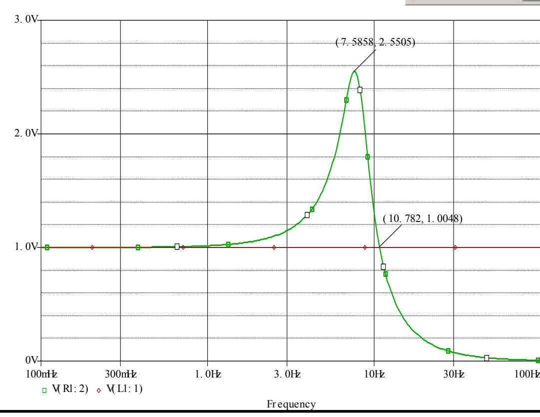



```
>> syms s
>> n=[0 0 2500];
>> dn=[1 20 2500];
>> g=tf(n,dn)

Transfer function:
      2500
-----------------
s^2 + 20 s + 2500

>> bode (g)
>> grid on
```

Matlab







Bode Diagram