ECE307-10

Active Filter Circuits

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Active Filter Circuits

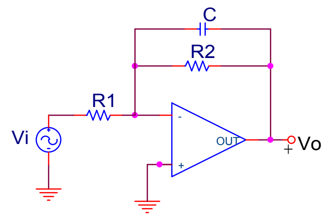
Introduction

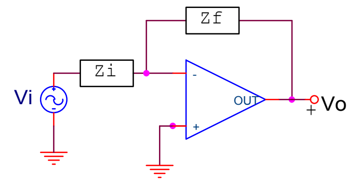
- Filter circuits with RLC are passive filter circuit
- Use op amp to have active filter circuit
- Active filter can produce band-pass and band-reject filter without using inductor.
- Passive filter incapable of amplification. Max gain is 1
- Active filter capable of amplification
- The cutoff frequency and band-pass magnitude of passive filter can change with additional load resistance
- This is not a case for active filters

We look at few active filter with op amps.
We look at that basic op amp filter circuits can be combined to active specific frequency response and to attain close to ideal filter response

Active Filter Circuits

First-Order Low-pass Filters





Transfer function of the circuit

$$H(s)=\frac{-Z_f}{Z_i}$$

$$H(s)=\frac{-R_2 \,||\, \frac{1}{SC}}{R_1}=\frac{-\frac{R_2}{sR_2C+1}}{R_1}$$

$$H(s)=\frac{-R_2}{R_1(sR_2C+1)}$$

$$H(s)=-K\frac{\omega_c}{(s+\omega_c)}$$

The Gain

$$K=\frac{R_2}{R_1}$$

Cutoff frequency

$$\omega_c=\frac{1}{R_2C}$$

Transfer function in jω

$$H(j\omega)=-K\frac{1}{(1+j\frac{\omega}{\omega_c})}$$

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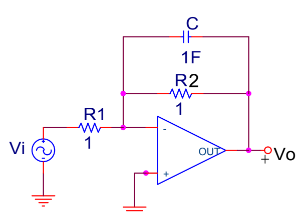
Example



- Find R_2 and C values in the following active Low-pass filter for gain of 1 and cutoff frequency of 1 rad/s.

From the gain

$$K=\frac{R_2}{R_1}=1$$

$$R_2=R_1=1\Omega$$

From the cutoff frequency

$$\omega_c=\frac{1}{R_2C}=1$$

$$C=\frac{1}{R_2}=1\,F$$

$$H(j\omega)=\frac{1}{(1+j\frac{\omega}{1})}$$

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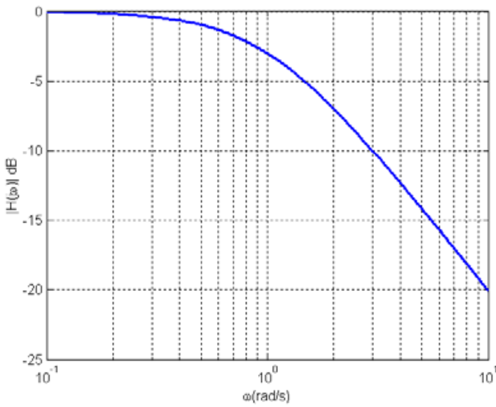
Example




```
>> w=0.1:.1:10;
>>
h=20*log10(abs(1./(1+j*w)))
;
>> semilogx(w,h)
>> grid on
>> xlabel('\omega(rad/s)')
>> ylabel('|H(j\omega)|
dB')
>>
```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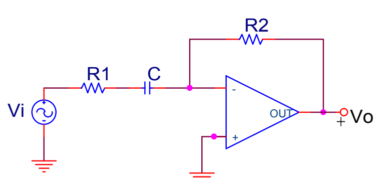
A first order high-pass filter



Transfer function of the circuit

$$H(s) = \frac{-Z_f}{Z_i}$$

$$H(s) = \frac{-R_2}{R_1 + \frac{1}{sC}} = \frac{-R_2 sC}{R_1 sC + 1}$$

$$H(s) = \frac{-R_2 s}{R_1(s + \frac{1}{R_1 C})}$$

$$H(s) = -K\frac{s}{(s + \omega_c)}$$

The Gain

$$K = \frac{R_2}{R_1}$$

Cutoff frequency

$$\omega_c = \frac{1}{R_1 C}$$

Transfer function in jω

$$H(j\omega) = -K\frac{\frac{j\omega}{\omega_c}}{(1 + j\frac{\omega}{\omega_c})}$$

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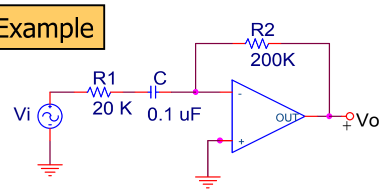
Example



- Find R_2 and R_1 values in the above active High-pass filter for gain of 10 and cutoff frequency of 500 rad/s.

From the cutoff frequency

$$\omega_c = \frac{1}{R_1 C} = 500 \qquad R_1 = \frac{1}{500C} = 20\,K\Omega$$

From the gain

$$K = \frac{R_2}{R_1} = 10 \qquad R_2 = R_1 10 = 200\ K\Omega$$

Transfer function in jω

$$H(j\omega) = -10 \frac{\frac{j\omega}{500}}{(1 + j\frac{\omega}{500})}$$

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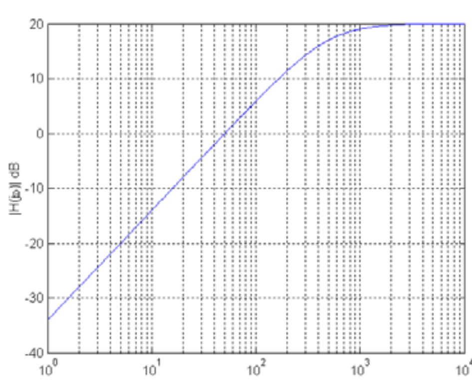
Example




```
>> w=1:10000;
>>
h=20*log10(10*(abs((j*w/500
)./(1+j*w/500))));
>> semilogx(w,h)
>> grid on
>> xlabel('\omega(rad/s)')
>> ylabel('|H(j\omega)|
dB')
>>
```

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Scaling

- In filter design, we can transform RLC values in to realistic values, this process is called scaling

- Two types of scaling, magnitude and frequency scaling

- In magnitude scaling, we multiply all L and R by scaling factor k_m, multiplying all C by $1/k_m$

$R' = k_m R$ $L' = k_m L$ $C' = \dfrac{C}{k_m}$

- k_m,is positive real number

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Scaling

- frequency scaling, we multiply all L, C by $1/k_f$ where k_f is scaling factor.

$R' = R$ $L' = \dfrac{L}{k_f}$ $C' = \dfrac{C}{k_f}$

- A circuit can be scaled in both magnitude and frequency in simultanously

$R' = k_m R$ $L' = \dfrac{k_m}{k_f} L$ $C' = \dfrac{C}{k_m k_f}$

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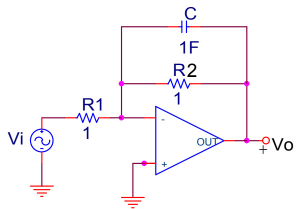
Example



- Example 1 , Find R_2 and R_1 values in the active Low-pass filter for gain of 5 and cutoff frequency of 1Khz and c=0.01 µF

$$k_f = \frac{\omega_c'}{\omega_c} = \frac{2\pi 1000}{1} = 6283.185$$

$$k_m = \frac{1}{k_f}\frac{C}{C'} = \frac{1}{6283.185(10^{-8})} = 15915.5$$

$$R_2' = k_m R_2 = 15915.5(1) = 15.9\ K\Omega$$

- For gain specification, we need to change R_1

$$R_1 = \frac{R_2}{K} = \frac{15.9K}{5} = 3.18\ K\Omega$$

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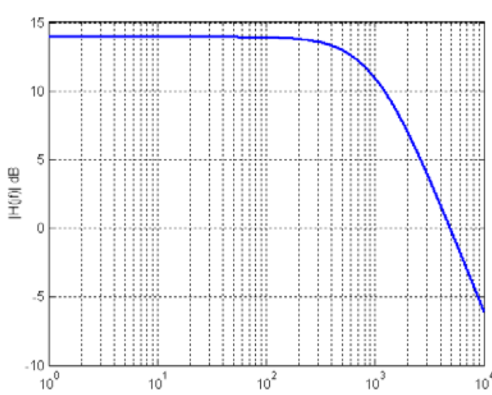
Example




```
>> f=1:10000;
>> w=2*pi*f;
>>
h=20*log10(5*abs(1./(1
+j*w/(2*pi*1000))));
>> semilogx(f,h)
>> grid on
>> xlabel('f(Hz)')
>> ylabel('|H(jf)|
dB')
```

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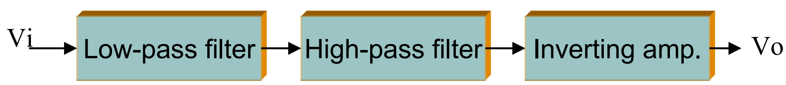
Op Amp Band-Pass Filters

- Three components
- A unity gain low-pass filter, cutoff frequency is ω_{c2}
- A unity gain high-pass filter , cutoff frequency ω_{c1}
- A gain component to provide the desired level

$$\frac{\omega_{c2}}{\omega_{c1}} \geq 2$$



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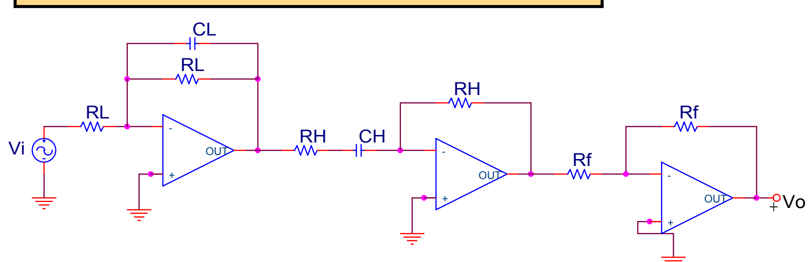
Op Amp Band-Pass Filters



$$H(s) = \left(\frac{-\omega_{c2}}{s+\omega_{c2}}\right)\left(\frac{-s}{s+\omega_{c1}}\right)\left(-\frac{R_f}{R_i}\right)$$

$$H(s) = \frac{-K\omega_{c2}s}{(s+\omega_{c2})(s+\omega_{c1})}$$

$$H(s) = \frac{-K\omega_{c2}s}{s^2+(\omega_{c1}+\omega_{c2})s+\omega_{c1}\omega_{c2}}$$

$$H(s) = \frac{\beta s}{s^2+\beta s+\omega_0^2}$$

$$\omega_{c2} \gg \omega_{c1}$$

$$\omega_{c2} = \frac{1}{R_L C_L}$$

$$\omega_{c1} = \frac{1}{R_H CH}$$

$$H(j\omega_0)\Big|_{max} = -K = -\frac{R_f}{R_i}$$

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Example:

- Design a band-pass filter for a graphical equalizer that has gain 2 within the frequency between 100 and 10,000 Hz. Use 0.1 μF capacitors

- For upper cutoff frequency from LP filter

$$\omega_{c2} = \frac{1}{R_L C_L} \qquad R_L = \frac{1}{\omega_{c2} C_L} = \frac{1}{2\pi 10000(0.1)10^{-6}} = 80\ \Omega$$

- For Lower cutoff frequency from HP filter

$$\omega_{c1} = \frac{1}{R_H CH} \qquad R_H = \frac{1}{\omega_{c1} C_L} = \frac{1}{2\pi 100(0.1)10^{-6}} = 7958\ \Omega$$

- For gain, choose R_i=1KΩ

$$K = \frac{R_f}{R_i} \qquad R_f = R_i K = 1000(2) = 2\ K\Omega$$

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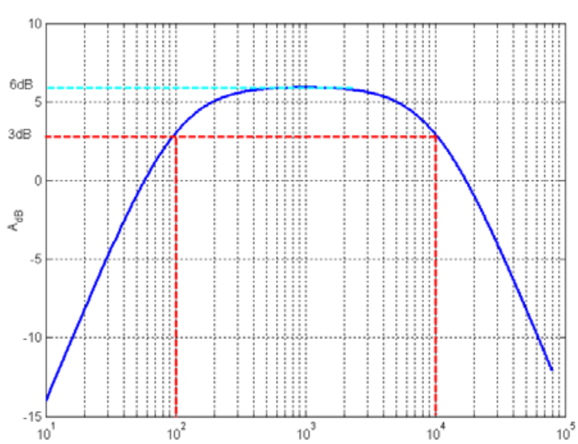
From transfer function

$$H(j\omega) = \left(\frac{-2\pi 1000}{j\omega + 2\pi 1000}\right)\left(\frac{-j\omega}{j\omega + 2\pi 100}\right)\left(-\frac{2000}{1000}\right) \qquad A_{dB} = 20\log_{10}|H(j\omega)|$$

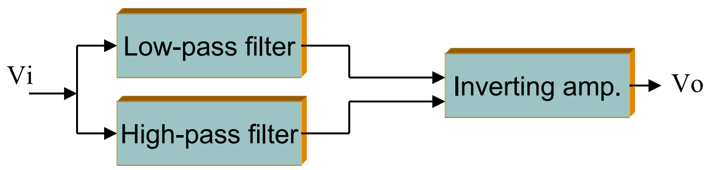



```
>> f=10:80000;
>> w=2*pi*f;
>> H=((-
2*pi*10000)./(j*w+2*pi*
10000)).*((-
j*w)./(j*w+2*pi*100))*(
-2);
>> A=20*log10(abs(H));
>> semilogx(f,A)
>> grid on;
>> ylabel ('A_{dB}')
>> xlabel ('F (Hz)')
```

Active Filter Circuits

Op Amp Band-Reject Filters

- Three components
- A unity gain low-pass filter, cutoff frequency is ω_{c1}
- A unity gain high-pass filter , cutoff frequency ω_{c2}
- A gain component to provide the desired level



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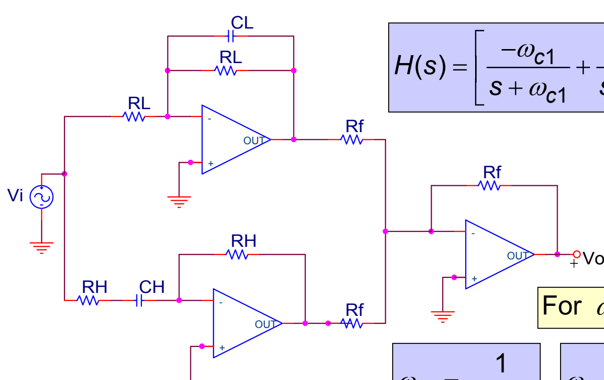
Op Amp Band-Reject Filters



$$H(s) = \left[\frac{-\omega_{c1}}{s+\omega_{c1}} + \frac{-s}{s+\omega_{c2}} \right] \left(-\frac{R_f}{R_i} \right)$$

For $\omega_{c2} >> \omega_{c1}$

$$\omega_{c1} = \frac{1}{R_L C_L} \qquad \omega_{c2} = \frac{1}{R_H C_H}$$

$$H(s) = \frac{R_f}{R_i} \left(\frac{s^2 + 2\omega_{c1} s + \omega_{c1}\omega_{c2}}{(s+\omega_{c1})(s+\omega_{c2})} \right)$$

$$H(j\omega)\Big|_{max} = K = \frac{R_f}{R_i}$$

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Example:

- Design an active band-reject filter that has gain 5 and the stop frequency between 100 and 2000 Hz. Use 0.5 μF capacitors

$F_{c1} = 100Hz \quad and \quad F_{c2} = 2000Hz$

For $\omega_{c2} >> \omega_{c1}$

$$\omega_{c1} = \frac{1}{R_L C_L}$$

$$R_L = \frac{1}{\omega_{c1} C_L} = \frac{1}{2\pi 100(0.5)10^{-6}} = 3.18\ K\Omega$$

$$\omega_{c2} = \frac{1}{R_H C_H}$$

$$R_H = \frac{1}{\omega_{c2} C_H} = \frac{1}{2\pi 2000(0.5)10^{-6}} = 159\ \Omega$$

- For gain, choose R_i=1KΩ

$$K = \frac{R_f}{R_i}$$

$$R_f = R_i K = 1000(5) = 5\ K\Omega$$

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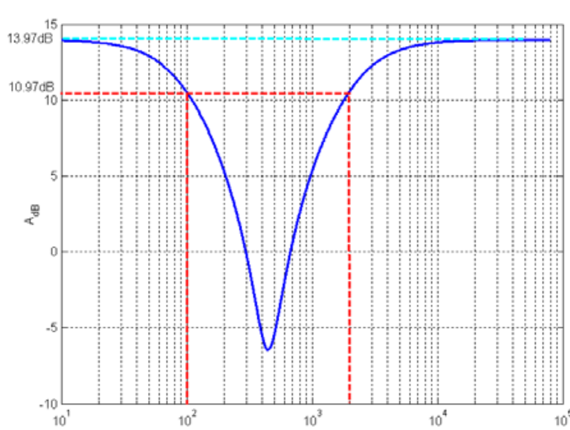
$$H(j\omega) = \left[\frac{-\omega_{c1}}{j\omega + \omega_{c1}} + \frac{-j\omega}{j\omega + \omega_{c2}}\right]\left(-\frac{R_f}{R_i}\right)$$

$$A_{dB} = 20\log_{10} |H(j\omega)|$$




```
>> f=10:80000;
>> w=2*pi*f;
>> H=(((-
2*pi*100)./(j*w+2*pi*100))
+((-
j*w)./(j*w+2*pi*2000)))*(-
5);
>> A=20*log10(abs(H));
>> semilogx(f,A)
>> grid on;
>> xlabel ('F (Hz)')
>> ylabel ('A_{dB}')
```