

Natural Radiation

There are a few radioisotopes that exist in our environment. Isotopes that were present when the earth was formed and isotopes that are continuously produced by cosmic rays can exist today if they have long enough half-lives. Here we will discuss 5 of these isotopes. Four were produced when the earth was formed: K^{40} (half-life of 1.277×10^9 years), U^{238} (half-life of 4.51×10^9 years), Th^{232} (half-life of 1.4×10^{10} years), and U^{235} (half-life of 7.1×10^8 years). C^{14} (half-life 5280 years) is continuously produced in the upper atmosphere by cosmic rays entering the earth.



Most of the potassium found on earth is the stable K^{39} . However, a small fraction, 0.0117%, of all potassium is K^{40} . K^{40} is radioactive with a half-life of 1.277×10^9 years. K^{40} decays in the following way: K^{40} has an 89.28% chance to undergo beta decay to the ground state of Ca^{40} , and a 10.72% chance to undergo electron capture to Ar^{40} . When electron capture occurs it is almost always to the excited state of Ar^{40} , 99.53% of the time. From the excited state, Ar^{40} decays to the ground state emitting a gamma particle with energy 1460.83 KeV. Thus when K^{40} decays, there is a 89.28% chance a beta particle is emitted and a $10.72(0.9953) = 10.67\%$ chance a gamma is emitted.

Although 0.0117% seems like a small amount, it means that one out of every 8500 potassium atoms is radioactive. This is a large amount! The average 70 Kg man has around 140 grams of potassium in his body. So the number of radioactive K^{40} nuclei in his body is:

$$\text{Number of } K^{40} \text{ nuclei} = \frac{140g}{40g} (6.02 \times 10^{23}) (0.000117) = 2.47 \times 10^{20} \quad (1)$$

which is a lot of nuclei. The activity due of K^{40} decays in the average person can be calculated using the relation: activity $A = N\lambda$, or $A = N(\ln 2)/\tau$:

$$A = \frac{2.47 \times 10^{20} \ln(2)}{1.277 \times 10^9 (365)(24)(3600) \text{sec}} = 4240 \frac{\text{decays}}{\text{sec}} \quad (2)$$

This amount of activity is equal to $4240/37000 = 0.11 \mu\text{Ci}$. Perhaps we should wear a radioactive sign around our necks! Fruits and vegetables can have as much as 0.4% potassium, and an average soil sample contains 2% potassium. Thus, one Kg of soil has an activity of $0.016 \mu\text{Ci}$ due to the potassium content alone. By measuring the gamma spectrum of soil and food samples for long times, one or two hours, the

potassium content can be measured to an accuracy of as good as 10%. K^{40} is the largest contributor to our natural background radiation.



Another isotope found in the earth is uranium 238. Due to its long half-life some still remains since the formation of the earth. U^{238} has a long decay series, undergoing alpha, beta and gamma decays until it finally becomes stable as lead 206, Pb^{206} . We list the complete U^{238} decay series:

Isotope	half-life	gamma energies
U^{238}	4.468×10^9 years	—
Th^{234}	24.1 days	63.3 (4.47%) 92.38 (2.60%) 92.80 (2.56%)
Pa^{234m}	1.17 minutes	765 (0.207%) 1001 (0.59%)
99.8% 0.13% Pa^{234}	6.75 hours	100 (50%) 700 (24%) 900 (70%)
U^{234}	2.47×10^5 years	53.2 (0.123%)
Th^{230}	8.0×10^4 years	67.7 (0.373%)
Ra^{226}	1602 years	186.2 (3.50%)
Rn^{222}	3.823 days	510 (0.076%)
Pc^{218}	3.05 minutes	—
99.98% 0.02% Pb^{214}	26.8 minutes	53.2 (1.1%) 242.0 (7.46%) 295.2 (19.2%) 351.9 (37.1%) 785.9 (1.09%)
At^{218}	2 seconds	—
Bi^{214}	19.7 minutes	609.3 (46.1%) 768.4 (4.89%) 806.2 (1.23%) 934.1 (3.16%) 1120.3 (15.0%) 1238.1 (5.92%) 1377.7 (4.02%) 1408.0 (2.48%) 1509.2 (2.19%) 1764.5 (15.9%)
99.98% 0.02% Po^{214}	164 microsec	806.2 (1.23%) 799 (0.014%)
Tl^{210}	1.3 minutes	296 (80%) 795 (100%) 1310 (21%)
Pb^{210}	21 years	46.5 (4.05%)
Bi^{210}	5.01 days	—
Po^{210}	138.4 days	803 (0.0011%)
Pb^{206}	Stable	

In the first column we list the isotopes in the decay series. If the decay is Alpha emission, the atomic number is lowered by 2 and the atomic mass is lowered by 4. For a Beta decay process, the atomic number increases by 1 and the atomic mass remains unchanged. In the middle column we list the half-life of the radioisotope shown in the first column.

In the last column we list the energy of the gamma rays emitted by the isotope in the first column. The number in parentheses is the yield percentage of the gamma. The isotopes in the series are called the daughter isotopes of U^{238} . From the table, one can see that as U^{238} decays via its daughter isotopes many alpha, beta and gamma particles are emitted.

Consider the following question: During the long decay series, how many of the various daughter isotopes are present at any one time? The half-lives are listed in the center column. Are there a smaller number of isotopes with short half-lives than isotopes with long half-lives at any particular moment? Any particular isotope will have a certain rate at which it is produced, and a rate at which it decays. Consider the situation in which isotope A decays to isotope B, which decays to isotope C. Let N_A be the number of nuclei of isotope A, N_B be the number of nuclei of isotope B, and N_C be the number of nuclei of isotope C. Let λ_A , λ_B , and λ_C be the corresponding decay constants for the decays. The rate at which the number of nuclei of isotope B decreases is $\lambda_B N_B$, which is just the decays/sec or the activity. The rate of formation of isotope B is just the decay rate of isotope A, $\lambda_A N_A$. So the change in the number of nuclei of isotope B per unit time is given by:

$$\text{Change of } N_B \text{ per unit time} = +\lambda_A N_A - \lambda_B N_B \quad (3)$$

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B \quad (4)$$

This rate-of-change equation applies to every radioactive nucleus in the decay series. The first nucleus, U^{238} , will only have the decay term $-\lambda N$, and the final nucleus, Pb^{206} , will only have the rate of formation term, $+\lambda N$. The solution to the series of differential equations is complicated. However, if we observe the decay series after a long time, long enough for the series to come into equilibrium, then the solution is simple. After a long time, the number of radioactive nuclei of a particular isotope remains constant in time, $dN_B/dt = 0$. The rate of formation, $+\lambda_A N_A$, is equal to the rate of decay, $-\lambda_B N_B$:

$$\lambda_A N_A = \lambda_B N_B \quad (5)$$

for every isotope in the decay series. This equilibrium condition is referred to as *secular equilibrium*. Since λN is equal to the activity of an isotope, for secular equilibrium the activity of each isotope in the series is the same. In terms of the half-lives of the isotopes, we have:

$$\frac{N_A}{\tau_A} = \frac{N_B}{\tau_B} = \frac{N_C}{\tau_C} \quad (6)$$

Isotopes with longer half-lives have more nuclei at any particular time in the decay series than isotopes with shorter half-lives.

In the experiment where we measure the spectrum of Brazil nuts, the decay series might not be in secular equilibrium. In fact, by measuring the counts under the appropriate photopeaks we can determine the age of the nuts. As before, suppose there are three isotopes A , B , and C , where $A \rightarrow B \rightarrow C$. If originally at $t = 0$ there were N_0 radioactive isotopes of A , then the number of A isotopes at time t is given by:

$$N_A = N_0 e^{-\lambda_A t} \quad (7)$$

Since $dN_B/dt = N_A \lambda_A - N_B \lambda_B$, we have

$$\frac{dN_B}{dt} = \lambda_A N_0 e^{-\lambda_A t} - \lambda_B N_B \quad (8)$$

The solution to this differential equation with the initial condition that $N_B(t = 0) = 0$ is (you should check the solution):

$$N_B = \frac{\lambda_A N_0}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t}) \quad (9)$$

Since the activity of B , $Act_B = \lambda_B N_B$ and the activity of A , $Act_A = \lambda_A N_A$ we have:

$$\frac{Act_B}{Act_A} = \left(\frac{\lambda_B}{\lambda_A}\right) \frac{\lambda_A}{\lambda_B - \lambda_A} (1 - e^{-(\lambda_B - \lambda_A)t}) \quad (10)$$

In terms of the half lives of the isotopes, $\tau = \ln 2 / \lambda$, the expression takes on the form:

$$\frac{Act_B}{Act_A} = \frac{\tau_A}{\tau_A - \tau_B} \left(1 - \left(\frac{1}{2}\right)^{\left(\frac{\tau_A - \tau_B}{\tau_A \tau_B}\right)t}\right) \quad (11)$$

How much time is needed for the series to reach secular equilibrium? The equation above describes the situation. If $\tau_A \gg \tau_B$, then the ratio of activities reduces to $1 - (1/2)^{t/\tau_B}$. If $t \gg \tau_B$ then the ratio of activities reduces to one and we have

secular equilibrium. For the U^{238} series, $\tau_A = 4.468 \times 10^9$ years. Since U^{234} has the longest half-life of the series of 2.47×10^5 years $\tau_A \gg \tau_B$. Also, since U^{238} was formed at a time much longer than this (at the earth's beginning), $t \gg \tau_B$ and we can assume that today the U^{238} series is at secular equilibrium.

However, for our Brazil nuts Ra^{228} is the "mother isotope" found in the Thorium series shown below. In this case $\tau_A = 6.7$ years and $\tau_B = 1.91$ years. In this case we have:

$$\frac{Act_B}{Act_A} \approx 1.4 \left(1 - \left(\frac{1}{2}\right)^{t/2.67}\right) \quad (12)$$

By measuring the ratio of activities we can determine the age of the Brazil nuts.

A good part of our natural background comes from the U^{238} decay series. One of the isotopes Rn^{222} is an inert gas. It can collect in rooms, and enter our lungs when we breath. When it decays, Pc^{218} is produced. Pc^{218} is not a gas, and can settle in the inner wall of the lungs. Having the remainder of the decay series take place inside the lungs can cause serious damage.

The largest peaks in the gamma spectrum from the U^{238} decay series are the 609 KeV (Bi^{214}), 352 KeV (Pb^{214}), 295 KeV (Pb^{214}) and 186 KeV (Ra^{226}). These peaks are easily identified in background spectra



Another isotope that makes up part of our natural background is Th^{232} . Below we list the complete th^{232} decay series:

Isotope	half-life	gamma energies
Th^{232}	1.405×10^{10} years	63.8 (0.267%)
Ra^{228}	6.7 years	—
Ac^{228}	6.13 hours	57.7 (0.487%) 99.5 (1.28%) 129.0 (2.42%) 154.0 (0.737%) 209.3 (3.88%) 270.2 (3.43%) 328.0 (2.95%) 338.3 (11.3%) 409.5 (1.94%) 463.0 (4.44%) 772 (1.50%) 794.9 (4.36%) 835.7 (1.61%) 911.2 (26.6%) 964.8 (5.11%) 969.0 (16.2%) 1588.2 (3.27%)
Th^{228}	1.91 years	84.4 (1.22%)
Ra^{224}	3.64 days	241.0 (3.97%)
Rn^{220}	55 seconds	550 (0.07%)
Po^{216}	0.15 seconds	—
Pb^{212}	10.64 hours	238.6 (43.6%) 300.0 (3.34%)
Bi^{212}	60.6 minutes	39.9 (1.1%) 727.3 (6.65%) 785.4 (1.72%) 1620.5 (2.32%)
64.06% Po^{212} 35.94% Tl^{208}	304 nsec 3.1 minutes	— 277.4 (6.31%) 510.77 (22.6%) 583.2 (84.5%) 763.1 (1.81%) 860.6 (12.4%)
Pb^{208}	stable	

Since the longest half-life other than the parent nucleus (Th^{232}) is 6.7 years for Ra^{228} , the series is in secular equilibrium today. The largest gamma peak of 239 KeV from Pb^{212} is easily observable in the background of spectra taken over a long time (one hour or more).

U^{235}

A very small part of natural radiation is from the U^{235} decay series. U^{235} is found in ores with U^{238} . On the average 0.7% of the uranium in ores is U^{235} . We list the complete U^{235} decay series below:

Isotope	half-life	gamma energy (KeV)
U^{235}	7.038×10^8 years	143.8 (10.96%) 163.33 (5.08%) 185.7 (57.2%) 205.3 (5.01%)
Th^{231}	25.5 hours	25.64 (14.5%) 84.2 (6.6%)
Pa^{231}	3.25×10^4 years	27.4 (10.3%) 300 (2.47%) 303 (2.87%)
Ac^{227}	21.6 years	70 (0.08%)
98.6% 1.4% Th^{227}	1.82 days	50.1 (8.0%) 236.0 (12.3%) 256.3 (7.0%) 300 (2.3%) 329.9 (2.7%)
Fr^{223}	22 minutes	50.1 (36%) 79.7 (9.1%) 234.8 (3%)
Ra^{223}	11.43 days	144.2 (3.22%) 154.2 (5.62%) 158.6 (0.69%) 269.5 (13.7%) 323.9 (3.93%) 338.3 (2.79%)
Rn^{219}	4 seconds	271.2 (10.8%) 401.8 (6.37%)
Po^{215}	1.78 millisecc	—
Pb^{211}	36.1 minutes	404.9 (3.78%) 427.1 (1.75%) 832.0 (3.52%)
Bi^{211}	2.15 minutes	351.1 (12.95%)
0.28% 99.7% Po^{211}	0.52 seconds	569.6 (0.0016%) 897.8 (0.26%)
Tl^{207}	4.79 minutes	897 (0.16%)
Pb^{207}	stable	

The series is at secular equilibrium today, since the longest half-life of the isotopes in the series is 3.25×10^4 years. U^{235} is an interesting isotope because of its fission properties. A single neutron can initiate fission. When U^{235} undergoes fission, it can release 3 neutrons. Each of these three neutrons can initiate another fission reaction, and a chain reaction can develop if the concentration of U^{235} is large enough. Because of its fission properties, U^{235} was the first radioisotope used for nuclear energy and weapons.

The peaks in the background gamma spectrum due to U^{235} are small and difficult to observe. However, with the high resolution of a Ge detector, the three gammas emitted by U^{235} can sometimes be seen in a uranium sample.

Laboratory Experiment on Natural Radiation

In this exercise you can choose from the following list of experiments on natural radiation:

1. Soil Sample Analysis: Your goal for this experiment is to measure the isotopes in a soil sample. You can pick a soil sample from the laboratory or your own soil sample. In most soils we can detect the following isotopes: the U^{238} series, the Th^{232} series, and K^{40} . Your task is to measure the amounts of the U^{238} and the Th^{232} isotopes, as well as any other isotopes you find. Since we know the natural abundance of K^{40} , you should also determine the amount of potassium in the soil. Express your results in percent by weight.
2. Natural Abundance of U^{235} in nature. Your goal for this experiment is to measure the natural abundance of U^{235} in various environmental samples.
3. Age of Brazil Nuts: Your goal for this experiment is to determine when a sample of Brazil Nuts fell from their tree. Brazil nuts bring up radium from the soil. They do not bring up very much radium, but enough for us to detect. The soil in which the trees grow in Brazil contains the Th^{232} decay series, so Ra^{228} is brought up into the nuts. By measuring the activity of the isotopes after Ra^{228} in the thorium series, you can determine when the nuts fell to the ground. This is a form of radioactive dating.
4. Geological Rock analysis: We have some geological rocks in the laboratory. You can measure the isotope composition of them. This is similar to the soil analysis.

We will record data from a number of samples in the lab, which include: a 24 hour background, energy calibration sources (Cs^{137} , Na^{22} , Co^{60}), soil samples, brazil nuts, KCl for efficiency calibration, pitchblend, fiestaware, radioactive glassware, and any samples that you bring in. You will need to use your experimental skills to best measure the desired quantities. I would like to let you figure things out on your own, like a real experimentalist, but since the laboratory time is short I list below some hints to help you in the experiments.

Hints in performing the Analysis

When you start to analyze the environmental samples, you will notice that there are many photopeaks to consider. Your first task is to determine how the energy of the gamma, E_γ is related to the channel number. For the Ge detector, the relationship between E_γ and channel number is remarkably linear. We will record data from some standard sources, Cs^{137} , Na^{22} , and Co^{60} , which you can use to find a relationship between channel number and energy. Once you have identified the energies of some of the photopeaks in your environmental sample, you can include these channel numbers in your fit for the energy calibration curve.

Most of the photopeaks are due to the main 2 or 3 decay series discussed above. In order to sort out the large amount of data, we find it useful to consider the ratio of (the gamma's detected/time)/(Yield in the decay series). In terms of the experimental parameters, the counting rate (i.e. the area/time under the photopeak) is equal to:

$$\frac{\gamma's\ detected}{time} = A\varepsilon Y \quad (13)$$

where A is the activity (in decays/sec) of the isotope, Y is the yield of the gamma, and ε is the efficiency for the source-detector geometry of the experiment. From this equation, one can see that (the gamma's detected/time)/(Yield) is

$$\frac{(\gamma's\ detected)/sec}{Yield} = A(Decays/sec)\varepsilon \quad (14)$$

For a decay series in secular equilibrium, each isotope in the series has the same activity!! Thus, if one plots the ratio (gamma's detected/sec)/(Yield) for all the different gammas in a particular decay series, the plot should be a smooth function of energy. The only energy dependence resides in ε , the efficiency of the detector. If one plots the ratio (gamma's detected/sec)/(Yield) for each gamma in the spectra, then each decay series present in the sample will have its own curve which varies smoothly with energy. From these curves, one can obtain the relative amounts of each decay series. For an absolute value, we will take data from a KCl sample that

has the same source-detector geometry as the environmental sample. Since we know the amount of K^{40} in the KCl sample, we can calibrate our environmental sample.

A nice property of the (gamma's detected/sec)/(Yield) plots is that the curves follow very closely to a power law relationship for gamma energies between 100 and 1500 KeV. Thus, a log-log plot of (gamma's detected/sec)/(Yield) for gammas between 100 and 1500 KeV enables an accurate determination of the isotope composition of the sample.

For samples that are not in secular equilibrium, a plot of (gamma's detected/sec)/(Yield) will not produce a smooth curve. Isotopes that have not reached secular equilibrium will have less activity than isotopes up the chain. This will result in a discontinuous plot. The amount of discontinuity can be used to determine the age of the sample.

Writeup for Experiment 5

1. (2 points) Turn in the data, a graph, and your best fit equation for the relationship between channel number and energy.
2. (4 points) Turn in the data and all your calculations for your determination of (gamma's detected/sec)/(Yield) for your sample.
3. (2 points) Turn in your graph(s) of (gamma's detected/sec)/(Yield) for your sample.
4. (4 points) Turn in the final results and analysis of the sample you choose. This should include your calculation for the efficiency of the detector if necessary.