Natural Abundance of K^{40} and Detector Efficiency

Your goal in this experiment is to determine the natural abundance of the isotope K^{40} in a KCl sample. The natural abundance is the fractional amount of the isotope compared to the non-radioactive isotopes found on earth. For K^{40}, the natural abundance is the ratio of the number of K^{40} nuclei divided by total number of non-radioactive potassium nuclei (mostly K^{39}) found on earth. Since isotopes are not separated by chemical means, the natural abundance of K^{40} is the same for any sample of potassium on earth. In this experiment, our sample will be KCl.

To find the natural abundance, we need examine a sample containing potassium, and measure

A) the total number of potassium (mostly K^{39}) nuclei, and
B) the total number of K^{40} nuclei.

A) We can determine the total number of potassium nuclei in our sample by measuring the mass of the KCl sample. Since we know the sample is pure KCl, the mass measurement gives us the number of moles. Using Avagadro's number, 6.02×10^{23}, you can determine the number of potassium nuclei in the sample.

B) Since $K40$ is radioactive, we can determine the number of K^{40} nuclei in the sample by measuring its activity. The number of radioactive nuclei, N, is related to the activity, A (decays/sec), by

$$A = \frac{Nln(2)}{\tau} \tag{1}$$

where τ is the half-life of the isotope. For K^{40} the half-life $\tau = 1.277 \times 10^9$ years. Once A is measured, the number of K^{40} nuclei N can be determined from the above equation. To obtain an accurate measurement of the activity, it is important to know the efficiency of the detector. The most important part of the experiment will be to determine the detector's efficiency. In the next section, we discuss what efficiency is and how to measure it.

Efficiency Calibration of Solid Scintillation Detectors

The efficiency ε of a detector is defined as (the number of particles detected)/ (the number of particles emitted):

$$\varepsilon = \frac{the\ number\ of\ particles\ detected}{the\ number\ of\ particles\ emitted} \tag{2}$$

The efficiency is a number between zero and one. If we know the efficiency of our detector, then measuring the number of particles detected will allow us to determine the number of particles emitted in our sample. The efficiency of a detector will depend on a few factors, the most important are:

1.**The source-detector geometry**: The number of particles detected will depend on how close the source is to the detector. The closer the source is to the detector, the larger the efficiency will be.

2.**The size of the detector**: Larger detectors will usually be more efficient, since they have a larger volume for the particles to absorbed in.

3.**The energy of the gamma (or X-ray) radiation**: The photopeak is produced by photo-absorption. The photo-absorption process has a strong energy dependence. For high energy photons, photo-absorption has a lower probability to occur than photons of low energy.

For solid scintillation detectors, NaI and Ge, the dependence of ε on energy, number 3 above, is quite large. For example, NaI detectors can detect 100 KeV gammas about 4-5 times more efficiently than 1200 KeV gammas. This means that although a photopeak at 1200 KeV is small compared to one at 100 KeV in a particular spectrum, there might be more 1200 KeV gamma emitted than 100 KeV gammas.

Since the efficiency depends on the three factors listed above, one often keeps the source-detector geometry fixed during a series of experiments. That is, for a series of experiments one places all the samples in the exact location relative to the detector. Also, one uses samples that are all the same size and shape. If this is done, then factors 1 and 2 above are the same for all the samples in a particular experiment. In this case, the only efficiency calibration necessary is the energy dependence of ε. The energy dependence for a particular source-detector geometry is measured by using standardized sources. One can purchase sources in which the activity has been calibrated by the manufacture. If the activity of the source is known, then the number of gamma particles emitted can be calculated. By measuring the number of gammas (of a particular energy) detected during a specific time interval, the efficiency ε can be determined.

An Example without including a Geometry Correction

As a sample efficiency calculation neglecting source-detector geometry, we consider the 662 KeV gamma emitted by Cs^{137}. Suppose the Cs^{137} source was 1.1μCi on May

20, 1990. This value is written on a calibrated sample. We place this source under our detector and record counts for 1 minute. Suppose the area under the 662 KeV photopeak is 20,000 counts. First one calculates the activity today, which is (say) May 20, 2004:

$$A = 1.1\mu Ci \frac{37000 decays/sec}{\mu Ci}(\frac{1}{2})^{(14years)/(30years)} \tag{3}$$

$$A = 30845\frac{decays}{sec} \tag{4}$$

since the half-life for Cs^{137} is 30 years, and 1μCi equals 37,000 decays/sec. Next one calculates the number of gammas that are emitted per second.

$$\gamma's\ emitted = 30845(0.85(\frac{\gamma's}{decay})) = 26200\frac{\gamma's}{sec} \tag{5}$$

since the gamma yield factor is 0.85. In one minute, (26218 gammas/sec)(60 sec/min) = $1.57x10^6$ gammas are emitted. The efficiency for the detector to detect a 662 KeV gamma is then

$$\varepsilon = \frac{20000 counts\ recorded}{1.57x10^6\gamma's\ emitted} = 0.0127 \tag{6}$$

The number 0.0127 is the efficiency of the detector for 662 KeV gamma particles for *the particular source-detector geometry* that we have used. Note that efficiency is unitless.

Efficiency Calculations including Source-Detector Geometry

The distance from the sample to the detector and the size of the detector are important factors that affect the detector's efficiency. We would like to include these affects in our measurements. The method that seems to work best (i.e. is simple and fairly accurate) with our NaI detectors is to use the ansatz:

$$\frac{Counts\ Detected}{time} = \frac{\gamma's\ emitted}{time}(\frac{\pi r^2}{4\pi(x+d)^2})\,\epsilon \tag{7}$$

where πr^2 is the cross sectional area of the detector, x is the distance from the source to the bottom of the detector, and d is the distance from the bottom of the detector to the "effective center" of the detector. The geometry factor $(\pi r^2)/(4\pi(x+d)^2)$ represents the fraction of gammas emitted that go through the detector. This geometry factor is just an approximation, but usually gives consistent results in our experiments. In our laboratory, we have NaI detectors of two sizes: with a diameter of 1 1/2 inches, and with a diameter of 2 inches. One can measure x, so once the "effective distance" d is known, the efficiency ϵ can be determined.

The number of γ's emitted can be determined from the activity, A, of the sample. For a particular γ energy, the number of γ's emitted per second equals the activity times the Yield, Y. The Yield is the probability that a γ is emitted during the nuclear decay. In terms of the activity and yield, the above equation becomes

$$\frac{Counts\ Detected}{sec} = AY(\frac{\pi r^2}{4\pi(x+d)^2})\,\epsilon \tag{8}$$

In the experiment, you will first determine d by collecting data from one source located at different distances x from the detector. A computer program that does a χ^2 fit of the data to determine the best estimate of d is available on the lab computers. The program was written by Sue Hoppe (2003), a Cal Poly Pomona physics major.

Once you have determined d for your detector, you can measure how the efficiency ϵ depends on the energy of the gamma. For standards, we will use Cs^{137} ($E_\gamma =$ 662KeV), Mn^{54} ($E_\gamma =$ 835KeV), and Na^{22} ($E_\gamma =$ 511 and 1275 KeV). Once the efficiencies for the energies of the calibration sources has been determined, a graph of efficiency vs. energy, $\epsilon(E)$ can be plotted. As you will see, ϵ has a strong energy dependence. There is no simple theory to use to determine what the shape of the calibration curve should be. Often one makes a log-log plot and assumes a power law relationship. Once the calibration curve has been determined, you can interpolate to find the efficiency of the detector for the energy of the gamma emitted by K^{40}.

Calibration of the efficiency graph is not as accurate as the calibration of energy for the scintillation detector. One problem is that it is expensive to obtain accurate calibrated standards. In our laboratory, the standards are calibrated for activity to within 5%. Errors also enter due to the uncertainty of the geometry factor and extrapolating. Thus the uncertainty in the efficiency calibration can be as large as 10-20%.

Experiment 3

Your goal in this experiment is to determine the natural abundance of K^{40} in a sample of KCl. The experiment consists of three parts: 1. determining the "effective distance" d for the geometry factor, 2. measuring different standards to obtain the energy dependence of the efficiency $\epsilon(E)$, and 3. measuring the KCl sample.

Measuring the Geometry Factor

In this part, you will measure one standard at 4 or 5 different distances from the detector. Place a calibrated source (i.e. Cs^{137}) in the top slot of your detector. Measure the distance from the source to the bottom of the detector, or some fixed point near the bottom of the detector. Record data for a specific time period. The time period will depend on the activity of the source. With a 1μCi Cs^{137} source you only need to take data for 1-2 minutes when the source is near the detector and 5 minutes when it is far away. Save your data and use the Gaussian curve fitting program to measure the area under the photopeak for each different distance setting. Your data table should look something like:

distance x	Counts Recorded	Counting Time	Counting Rate (1/sec)
—	—	—	—

Use the computer program on your lab computer to fit the data and determine the "best fit" value for the effective distance d.

Measuring the Energy Dependence of ϵ

Using the calibrated sources Cs^{137}, Na^{22}, and Mn^{54}, determine the efficiency ϵ for the energies of the photopeaks. For best accuracy, you should place all the sources at approximately the same distance from the detector. Choose a distance that is close and is approximately the distance where you will place the KCl sample. Use the Gaussian curvefitting program to determine the counts under the photopeak (area).

Data for Standards

Isotope	half-life	Energy	Yield
Cs^{137}	30 yrs	662 KeV	0.85
Na^{22}	2.62 yrs	511 KeV	1.80
Na^{22}	2.62 yrs	1275 KeV	1.0
Mn^{54}	302 days	835 KeV	1.0

Data for K^{40}

Isotope	half-life	Energy	Yield
K^{40}	1.277×10^9 yrs	1460 KeV	0.1069

Once you have calculated $\epsilon(E)$ for the 4 standard energies, graph your results. Extrapolate your graph to estimate the efficiency ϵ for the energy of the gamma given off by K^{40} (i.e. 1460 KeV).

Measuring the KCl sample

Obtain a planchet from the instructor and measure its mass. Fill the planchet with KCl and measure the mass to obtain the mass of the KCl only. Place the KCl sample as close as you can to the detector. Measure the average distance to the detector. Record data for an hour. Since there is just a little amount of radiation emitted from the sample, we need long counting times to get good statistics.

After the counting period is finished, use the Gaussian curvefitting program to measure the counts under the $KCl\ -\gamma$ photopeak (1460 KeV). Before class, the instructor started a long background measurment. Measure the counts under the $KCl\ -\gamma$ photopeak for the background. Carry out the necessary calculations to determine the natural abundance of K^{40} in your sample. Estimate the uncertainty of your value.

Laboratory Writeup for Experiment 3

1. (2 points) Turn in your data and results for measuring the "effective distance" d.

2. (4 points) Turn in all your data and all calculations for measuring the efficiency ϵ for each of the 4 calibration energies.

3. (2 points) Turn in a graph of ϵ versus E for the 4 calibration energies, and explain how you extrapolated to find $\epsilon(1460\,KeV)$.

4. (3 points) Show all calculations for your estimate of the natural abundance of K^{40} in your KCl sample.

5. (1 point) Question: If a 1000 KeV gamma ray interacts with the NaI detector. Which process (photoelectric effect or Compton Scattering) is most likely to occur? Why?