PHYSICS 403
QUANTUM MECHANICS
SECOND EXAM SPRING 2008
Instructor: Peter Siegel

Instructions: This is a take home exam. You are to work on the problems by yourself. Since partial credit is given, show all your work. The exam is due in class on Wednesday May 21. There are 3 problems for a point total of 30.

Problem 1. (10 points) Wolfram will be helping his father at Los Alamos do a pion scattering experiment off a boron target. The experimental setup is shown on the figures page at the end.

The incident pion flux is 10^8 pions/sec/cm^2, and the target contains 10^{18} boron nuclei. The detector, which has an area of 3 cm^2, is located 1 meter from the target and is at an angle of 30 degrees from the incident direction.

A theorist has calculated the elastic scattering amplitude to be:

$$f(\theta) = \frac{i}{2k}(1 + 3cos\theta) \tag{1}$$

where k is the wave number, $k = p/\hbar$. The experiment is carried out with pions having a wave number of $k = 0.5$ fm^{-1}. Note 1 fm $= 10^{-13}$cm.

a) What is the differential cross section, $d\sigma/d\Omega$, for an arbitrary angle θ in units of milli-barns? (Note: 1 fm^2 = 10 milli-barns).

b) How long must Wolfram wait before he detects 100 pions in his detector? (Note: this will give him a 10% statistical uncertainty.)

c) What is the total elastic cross section?

d) The optical theorum states that the total cross section is equal to $\sigma = (4\pi/k)Im(f(0°))$. Determine the total cross section using the optical theorum. This answer will not agree with your result from part c. I'll explain why in lecture.

Problem 2. (10 points) A particle of mass m and kinetic energy E is incident upon a scattering target. The potential describing the interaction is shown on the figures page.

The energy of the particle E is less than V_0.

Find an expression for the l=0 phase shift for this interaction. You can express your answer using trig functions (sin, cos, tan) and/or hyperbolic functions (sinh, cosh, tanh).

Problem 3. (10 points) Consider scattering from a Dirac delta-shell potential. The potential can be written:

$$V(r) = \alpha\delta(r - a) \tag{2}$$

See the figures page.

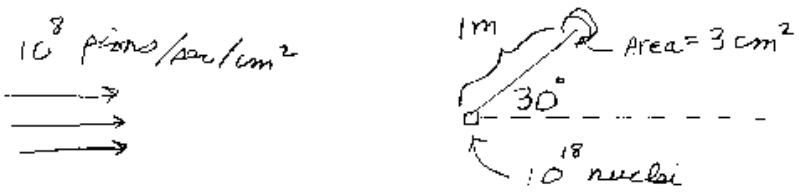
The incident particle has momentum $p = \hbar k$.

a) Use the Born approximation to calculate the differential cross section.

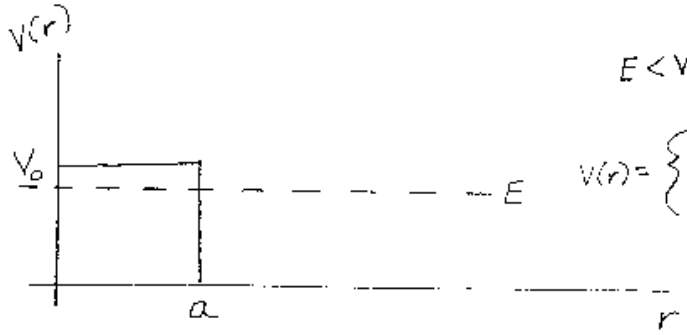
b) Make a sketch of the differential cross section as a function of scattering angle θ.

c) For what value of theta does the differential cross section have its first minimum? Express your answer in terms of k and a.

(1)



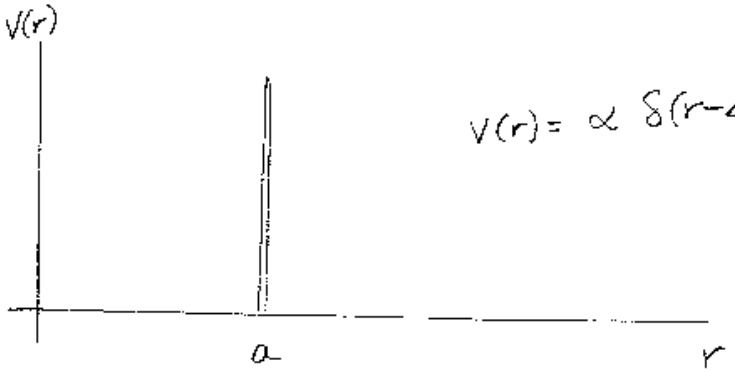
(2)



$E < V_0$

$$V(r) = \begin{cases} V_0 & r \le a \\ 0 & r > a \end{cases}$$

(3)



$V(r) = \alpha\, \delta(r-a)$