PHYSICS 403
FIRST EXAM SPRING 2008
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Instructions: This is a take home exam. You are to work on the problems by yourself. Since partial credit is given, show all your work. The exam is due in class on Monday April 28. There are 3 problems for a point total of 30.

Problem 1. (12 points)
A particle of mass m is trapped in a one-dimensional infinite square well of length a. It starts off in the ground state at $t = -\infty$:

$$\psi(x, t = -\infty) = \sqrt{\frac{2}{a}} sin(\frac{\pi x}{a}) \tag{1}$$

The particle is subjected to the following time dependent interaction:

$$\hat{H}'(x, t) = V_0 a \delta(x - a/2) e^{-(t/\tau)^2} \tag{2}$$

Where V_0 has units of energy, τ has units of time, and $\delta(x - a/2)$ is the Dirac delta function.

What is the probability that the interaction causes the particle to be excited to the n'th excited state after a long long time (i.e. at $t = +\infty$)?

Problem 2. (10 points)
A particle of mass m and charge e is confined to move along the z-axis in a harmonic oscillator potential. The energies of the particle are quantized as

$$E_n = (n + 1/2)\hbar\omega \tag{3}$$

At t = 0 it is in the first excited state (n = 1). Since the particle has charge, it will decay spontaneously.

Determine an expression for the probability (per unit time) for the particle to decay to the ground state (n = 0). Hint: since the state function is only on the z axis, the matrix element $< \psi_f|\vec{r}|\psi_i >$ is just $< \psi_f|\hat{z}|\psi_i >$. The $\hat{z}$ operator can be expressed in terms of the raising and lowering operators for the harmonic oscillator.

Note: This problem is applicable to the vibrational transitions of a diatomic molecule which has an electric dipole moment, i.e. HCl.

Problem 3. (8 points)
Consider a two-state quantum system. The following Hamiltonian describes the dynamics of the system:

$$\hat{H} = \hat{H}_0 + \hat{H}'(t) \tag{4}$$

where $\hat{H}'(t)$ is "smaller" than $\hat{H}_0$ and depends on time, whereas $\hat{H}_0$ does not depend on time. As done in class, we choose as a basis for our analysis the eigenstates of $\hat{H}_0$: labeled $|a>$ and $|b>$, with energies E_a and E_b. $\hat{H}_0|a>= E_a|a>$ and $\hat{H}_0|b>= E_b|b>$.

As discussed in class, at anytime t, the state of the system ψ can be expressed in terms of these eigenstates:

$$|\psi>= c_a(t)|a> + c_b(t)|b> \tag{5}$$

Show that $<\psi|\psi>$ does not change in time. (Hint: show that

$$\frac{d}{dt}(c_a^* c_a + c_b^* c_b) = 0 \tag{6}$$