

Phy403 Quantum Mechanics
Fourth Homework Assignment
Due Friday May 30

Problem 1. In class we found the solution to the Dirac equation for a free particle of momentum p . What is the solution to the Dirac equation for a free anti-particle of momentum p ?

Problem 2. Show that:

$$\vec{\alpha} \times \vec{\alpha} = \frac{4i\vec{S}}{\hbar} \quad (1)$$

where \vec{S} is the spin matrix defined in class.

Problem 3. Show that the relativistic expression for the probability flux reduces to the non-relativistic expression:

$$\frac{-i\hbar}{2m}(\psi^* \nabla \psi - \psi \nabla \psi^*) \quad (2)$$

Problem 4. Using the explicit forms of the Pauli matrices, show that the following relationships are true:

$$\begin{aligned} [\sigma_i, \sigma_j] &= 2i\epsilon_{ijk}\sigma_k \\ \{\sigma_i, \sigma_j\} &= 2I\delta_{ij} \\ \sigma_i\sigma_j &= \delta_{ij} + i\epsilon_{ijk}\sigma_k \\ (\vec{\sigma} \cdot \vec{a})(\vec{\sigma} \cdot \vec{b}) &= \vec{a} \cdot \vec{b}I + i\vec{\sigma} \cdot (\vec{a} \times \vec{b}) \end{aligned}$$