Geiger Counter Experiments

In this laboratory session, we will do different experiments with the Geiger Counter: determining the proper operating voltage, the efficiency, the dead time, investigating the probabilistic nature of the decay and measuring the half-life of Ba^{137m}.

1. Operating Voltage of the Geiger Counter

First, we will measure the Geiger counter's response as a function of applied voltage.

a) Place a source under the Geiger counter tube.

b) Set the timer to count for ten minutes or longer.

c) Set the voltage to zero first, then slowly turn up the voltage until the counter starts to record counts. This is the "starting voltage".

d) Take 1 minute readings, increasing the voltage by about 10 or 20 volts each time. Make a table of your data.

Note: to prevent damaging the tube, **do not increase the voltage more than 150 volts beyond the starting voltage, and certainly not more than 1000 volts.**

e) Graph your results using Excel or on linear graph paper. Label on your graph the starting voltage and the plateau region. Also label the proper operating voltage on the graph. From your graph, do you think your Geiger counter tube is operating properly? Why or why not?

2. Efficiency of the Geiger Counter

In this part, you will estimate the efficiency of the Geiger-Mueller tube for a particular source. The efficiency of the Geiger counter will depend on the sample, so be sure to record the sample used. From the activity written on the source, use the half-life formula to determine the activity in decays/sec of your source today. Place your source as close to the tube as you can, and count for 2 minutes. Estimate the distance the source is away from the tube. Determine the efficiency of the Geiger-Mueller tube for this positioning of the source. We will define the efficiency ϵ as:

$$\epsilon \equiv \frac{particles\ detected}{particles\ emitted} \tag{1}$$

3. Examining the Geiger Counter Time Series

For this exercise we will record the time between signals from the Geiger Tube. There is one Geiger Tube in the room that is connected to a computer for the time measurements. We will record around 50000 times. The times are in units of micro-seconds (μsec), 1×10^{-6} sec.

1. Obtain the data file from the instructor, and import the 50000 times into Excel.

2. Using the analysis tools, make a histogram of the number of times within a certain range and bin size. For example you might try a bin size of 100 μsec.

3. From your frequency graph, estimate the dead-time of the detector.

4. Try and fit your frequency graph with an exponential function. How good is the fit? What property of nature does this data support? See the theory section at the end.

4. Measurement of the half-life of Ba^{137m}.

The half-life of Ba^{137m} is on the order of minutes. In this experiment, we will record the counts for the Ba^{137m} source for a 10 second counting time. We take these readings every 30 seconds. Before you start the experiments make a data table simiiliar to the form below:

time (sec)	Counts in 10 seconds
0	...
30	...
60	...
90	...
...	...

1. After the instructor places the sample under your Geiger Counter tube, start recording data.

2. After correcting for dead time and background, make a graph of the number of counts/sec as a function of time. Fit the graph with an exponential function. Determine the decay constant (and half-life) from the slope of the graph.

Report for Experiment 1

1. Make a table and graph of Counts vs. voltage for your Geiger counter tube. Label on your graph the starting voltage, operating voltage, and the plateau region.

2. Show your data and calculations for determining the efficiency of your Geiger counter.

3. Make a frequency graph of the time between Geiger tube signals. From the graph estimate the dead time of the Geiger counter in micro-seconds. Is the frequency graph an exponential function? What property of nature does your data support?

4. For the Ba^{137m} decay, use Excel to make a graph of counts/sec vs. time. Include the corrections for background and dead-time.
a) Is the decay exponential? i.e. does it obey the half-life formula?
b) If it does follow an exponential decay, what is the half-life of the decay?

Geiger Counter Time Series

In the third section, we measured the times between successive recordings of the Geiger Counter. The times appear to be random, and if one were to check for randomness, one would find that they are truly random. We also plotted a frequency plot (or histogram) of these times. The frequency plot appears to be an exponential function. This dependence can be understood from one principle of nature:

Each radioactive nucleus has a certain probability to decay per unit time. This probability does not depend on how long the nucleus has been in its excited state (i.e. radioactive).

The probability to decay per unit time is denoted by the symbol λ, and is called the decay constant of the decay. λ has units of 1/time. The probability that a particular nucleus will decay in the time interval δ is $\lambda\delta$ **in the limit as** $\delta \to 0$. Our radioactive sample has a large number of radioactive nuclei, N_0. The probability that one nucleus will decay in the time interval δ is $N_0\lambda\delta$ in the limit as $\delta \to 0$. If the efficiency of our detector is ϵ, then the probability that our Geiger Counter tube will detect a particle in the time interval δ is $\epsilon N_0 \lambda \delta$, in the limit as $\delta \to 0$. For convenience we define $A \equiv \epsilon N_0 \lambda$. If the decay process is probabilistic, then there exists a probability per unit time A such that the probability that our Geiger Counter tube will detect a particle in the time interval δ is $A\delta$, in the limit as $\delta \to 0$.

Now consider our experiment. Since N_0 is very large, it hardly changes during our experiment. So A is essentially the same for every recording. We can ask the following question: What is the probability P_{not} that we will not record a count within a time t since the last count was recorded? This can be answered as follows. Divide the time t into N equal segments, each of duration δ. That is $\delta = t/N$. Then P_{not} is given by

$$P_{not} = (1 - A\delta)^N \tag{2}$$

Now we need to take the limit as $\delta \to 0$, or as $N \to \infty$:

$$P_{not} = \lim_{N\to\infty} (1 - \frac{At}{N})^N \tag{3}$$

This limit is the exponential to the base e:

$$P_{not} = e^{-At} \tag{4}$$

In our histogram, we have binned the data in intervals of Δt. The probability that the Geiger counter will receive a signal between the time t and $t + \Delta t$ is therefore:

$$P(t)\Delta t \approx e^{-At} A \Delta t \tag{5}$$

We have used the approximation sign, since A is only defined in the limit as $\delta \to 0$ and Δt is a finite time. If we collect a total of C_{tot} data points, then the number of counts recorded between time t and $t + \Delta t$ is just

$$C(t) \approx C_{tot} A e^{-At} \Delta t \tag{6}$$

As $C_{tot} \to \infty$ and $\Delta t \to 0$ the approximation gets better. Your data are consistent with the probability hypothesis if $C(t)$ decreases exponentially.

Half-Life Decay

This experiment is somewhat similar to the time series analysis. We will be recording the times between successive readings of our Geiger Counter. In this case, however, we will be recording data from an isotope that has a short half-life. The total number of radioactive nuclei will decrease throughout the experiment. Suppose we start the experiment at time $t = 0$. Let $N(t)$ be the **average number** of radioactive nuclei present after time t. As before, the probability that one nucleus will decay in a time δ is $\lambda\delta$ in the limit as $\delta \to 0$. In a small time increment δ, the average number of nuclei that decay is:

$$N(t) - N(t+\delta) \approx N(t)\lambda\delta \tag{7}$$

The approximation becomes better as $\delta \to 0$. Taking this limit we have

$$\frac{dN(t)}{dt} = -\lambda N(t) \tag{8}$$

The solution to this equation is

$$N(t) = N_0 e^{-\lambda t} \tag{9}$$

Remember that this equation is for the expected average number of radioactive nuclei, $N(t)$, present at time t. If $N(t)$ is the actual number of radioactive nuclei, then the equation is only an approximation. Since then the left side is an integer, and the right side a continuous function of t.

In our experiment, we will record the times, starting from $t = 0$, when the counter receives signals. We will then determine the number of counts received between (say) $t = 0$ and $t = 10$sec, $t = 10$sec and $t = 20$sec, etc... Call the number of counts received between time t and $t + \Delta t$ as $D(t)$ where Δt could be 10 seconds. Then,

$$D(t) \approx \epsilon N(t)\lambda\Delta t \tag{10}$$

where ϵ is the efficiency of the detector. The approximation is due to two reasons. First, Δt is a finite time. Second, $N(t)$ is an average value, so the above equation represents the expected value for $D(t)$. If we substitute in for $N(t)$ we have:

$$D(t) \approx (\epsilon N_0 \lambda \Delta t)e^{-\lambda t} \tag{11}$$

Thus, we see that $D(t)$ should decrease (approximately) exponentially, with the approximation getting better as Δt gets smaller. One can also cast this equation in terms of base 2:

$$D(t) \approx (\epsilon N_0 \lambda \Delta t)(\frac{1}{2})^{t/\tau} \tag{12}$$

where τ is called the half-life. Equating the two equations gives a relationship between λ and τ:

$$\tau = \frac{ln(2)}{\lambda} \tag{13}$$