VECTOR ANALYSIS & COMPLEX VARIABLES
EGR 512
Summer 2000
Problem Set 7
Potential Flow Theory

Problem 1) The steady flow of a fluid through a 3-D region is described by the velocity field

$$\vec{v} = (Ax + y)\hat{i} + (Bx + 2y + Cz)\hat{j} + (Dx + 3y - 3z)\hat{k}$$

(a) Given that this flow field is incompressible and irrotational, determine the four unspecified constants *A, B, C* and *D*.

(b) If the fluid temperature is given by the following temperature field:

$$T(x,y,z) = exp(-0.1z)[100 - (x^2 + y^2)]$$

determine the rate of change of temperature experienced by the fluid particle passing through the point $(x,y,z) = (1,1,0)$.

Problem 2) Consider the complex potential

$$F(z) = V_o\left[z + y_o\, exp(i2\pi\ z/\lambda)\right]$$

where V_o, y_o and λ are constants.

(a) Show that the stream function for this complex potential is given by:

$$\psi = V_o\left[y + y_o\, exp(-2\pi y/\lambda) sin\left(\frac{2\pi x}{\lambda}\right)\right]$$

(b) Confirm that for large values of y, $\psi = V_o\, y$, which is the stream function of a uniform flow of velocity parallel to the x-axis.

(c) Verify that by expanding the exponential term of part (a) and considering the limit where the ratio $2\,\pi y/\lambda << 1$, one arrives at:

$$\psi = V_o\left[y + y_o\, sin\left(\frac{2\pi x}{\lambda}\right)\right]$$

(d) Finally, using the result form part (c), show that the $\psi = 0$ streamline is given by

$$y = -y_o \sin\left(\frac{2\pi x}{\lambda}\right)$$

and the function ψ may therefore be regarded as representing the flow over a sinusoidally corrugated wall, where y_o is the amplitude of the corrugation and λ is the wavelength as shown below.

<u>Problem 3)</u> Consider 2-D potential flow over a body which has a constant velocity (U,V) at infinity. Then the complex potential is

$$w(z) = (U - iV)z + f(z)$$

where df/dz vanishes at infinity. If we take the origin to be inside of the body, then the boundary conditions is $\psi = Im[w(z)] = 0$ on the body surface. This condition can be satisfied for any boundary not passing through the origin by a series of the form

$$\sum_{n=1}^{\infty} \frac{A_n}{z^n}$$

where the An are complex constants $a_n + ib_n$. We further allow clockwise circulation of strength Γ around the body and source flux of strength Q from the body, both centered on the origin.

(a) Write down the total complex potential for this flow over an arbitrarily shaped body.

(b) Use Blausius' Theorem to compute the forces on the body in terms of U, V, Q, Γ, the a_n and b_n.

(c) Resolve the X, Y forces form part (b) into lift L and drag D. (Recall, Drag is the force along the direction of the total velocity vector. Lift is the force normal to the total velocity vector and is rotated $\pi/2$ CCW from the drag)