**VECTOR ANALYSIS & COMPLEX VARIABLES**
**EGR 512**
**Summer 2000**
**Problem Set 1**
**Vector Algebra**
**<u>Due 6/26/00</u>**

**<u>Problem 1)</u>** Referring to the figure below, derive the formulae describing the transformation from one rectangular coordinate system ($K$ to $K'$) to another given by:

$$x_l = \alpha_{k'l} x'_k + x'_{0l}$$
$$x'_l = \alpha_{l'k} x_k + x_{0l}$$

The transformation coefficients $\alpha$ of the above expressions satisfy certain orthogonality conditions, which can be obtained by expanding the basis vectors of system $K$ with respect to the basis vectors of the system $K'$, and vice versa. Show that these orhtogonality conditions are given by:

$$\alpha_{l'k}\alpha_{l'm} = \delta_{km}$$
$$\alpha_{k'l}\alpha_{m'l} = \delta'_{km}$$

where the Kronecker delta

$$\delta_{km} = \hat{i}_k \bullet \hat{i}_m = \begin{cases} 0 & \text{if } k \neq m \\ 1 & \text{if } k = m \end{cases}$$
$$\delta'_{km} = \hat{i}'_k \bullet \hat{i}'_m = \begin{cases} 0 & \text{if } k \neq m \\ 1 & \text{if } k = m \end{cases}$$

has been introduced.

**Problem 2)** (a) Use vectors to derive the law of cosines: $c^2 = a^2 + b^2 - 2ab\cos\alpha$

(b) Express the scalar (dot) product of two vectors in terms of their covariant and contravariant components.

(c) Find the vector (cross) product of two vectors in an oblique coordinate system.

**Problem 3)** Show that the trajectory of a particle moving under the influence of gravitational attraction is a conic section if the equation of motion of the particle is of the form

$$\frac{d\vec{v}}{dt} = -\alpha\frac{\vec{r}}{r^3} = -\alpha\frac{\vec{r}_o}{r^2}$$

where $\alpha$ is a constant and

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{r}_o = \frac{\vec{r}}{r}$$

are the velocity and unit radius vectors, respectively.