

PLEASE TURN OFF ALL CELL PHONES AND BEEPERS. Do the following problems, BOXING IN all answers. An 8.5" x 11" sheet of notes (2 sides) and a calculator may be used. Cell phone calculators, PDAs, and all transmitting/receiving devices are forbidden. Each problem labeled (a) or (b) carries the same weight. Please turn in this exam with your solution pages.

1. A projectile of mass m and initial velocity \vec{v}_o , making angle $\theta \leq \pi/2$ with the positive x-axis, is launched from ground level at $x = 0, y = 0$. Two forces act on the projectile: the weight force $\vec{W} = -mg\hat{y}$ in the negative y direction, and a constant force $\vec{F} = -b\hat{x}$ in the negative x direction.

(a) Find the horizontal range R as a function of b, m, g, v_o , and θ .

(b) In the special case for which $b = mg$, for what two values of θ is $R = 0$? For both θ values, sketch y vs. x and describe the physics in words.

2. At time $t = 0$, a body of mass $m = 2$ kg enters a viscous medium at $x = 0$ with velocity $10\hat{x}$ m/s. The net force on it is $F_{net} = -\{3v(x) + [v(x)]^2\}\hat{x}$, where $v(x)\hat{x}$ is the velocity.

(a) Find $v(x)$ as a function of x .

(b) At what time t does the body have speed 5 m/s? Note: $\int \frac{dx}{a+be^{px}} = \frac{x}{a} - \frac{1}{ap} \ln(a + be^{px}) + constant$

3. A sphere of mass $m = 1$ kg moves in one dimension. When it's at position x , its potential energy is $V(x) = \cosh(2x) - \sinh(3x)$, where $\cosh(y) \equiv \frac{1}{2}[e^y + e^{-y}]$ and $\sinh(y) \equiv \frac{1}{2}[e^y - e^{-y}]$.

(a) Find the sphere's equilibrium position and the angular frequency ω of its small oscillations.

(b) The sphere is moved away from equilibrium and released from rest. It undergoes small oscillations with damping force $-b\dot{x}(t)$. If the oscillation amplitude decreases by half in 6.00 seconds, what is b ?

4. An object of mass $m = 1 \times 10^3$ kg is fired directly toward the moon with launch speed v_o . Earth's mass is M_E , Earth's radius is R_E , the moon's mass is M_L , and the moon and Earth centers are separated by distance d . Assume that both Earth and moon are at rest. Note: $M_E = 5.97 \times 10^{24}$ kg, $R_E = 6.37 \times 10^6$ m, $M_L = M_E/81.6$, $d = 3.84 \times 10^8$ m, $G = 6.67 \times 10^{-11}$ N · m²/kg².

(a) Denote the object's distance from Earth's center by x . For what value of x does the gravitational potential energy $V(x)$ of the Earth+moon+object system attain its maximum value?

(b) For what initial speed v_o will the object travel to $x = d/2$, stop, and reverse direction?