"Sledge-Hammer Technique"

Given: $\dot{v}_x = \omega_c v_y$ and $\dot{v}_y = -\omega_c v_x$ with $\omega_c \equiv qB/m$, define $\eta \equiv v_x + iv_y$. Differentiate η and use the latter two equations to obtain $\dot{v}_x + i\dot{v}_y = -i\omega_c$. The solution of this is $\eta = A_c exp(-i\omega_c t)$, where AO_c is complex in general. Write $A_c = Aexp(i\delta)$, where A and δ are real. It follows that

$$\eta = Ae^{i(\delta - \omega_c t)}$$

.

Now integrate η two ways. The first way is:

$$\int \eta \, dt = x(t) + iy(t) + C_1. \quad (1)$$

Here C_1 is complex in general.

The second way is

$$\int \eta \, dt = \int Ae^{i(\delta - \omega_c t)} \, dt = \frac{Ae^{i\delta}}{-i\omega_c} e^{-i\omega_c} t + C_2 \equiv C_3 e^{-i\omega_c t} + C_2 \quad (2)$$

The last step defines the (complex) C_3.

Equating Eqs. (1) and (2), we obtain

$$x + iy = C_3 e^{i\omega_c t} + [C_2 - C_1] \quad (3)$$

We then define $C_2 - C_1 \equiv X + iY$, with X and Y real. Note that for $t = 0$ we obtain $C_3 = [x(0) - X] + i[y(0) - Y]$. We can also write

$$[x - X] + i[y - Y] = C_3 e^{-i\omega_c t}. \quad (4)$$

Here C_3 is a complex constant.

Now, let us rewrite Eq. (4) as

$$[x - X] + i[y - Y] = \{[x(0) - X] + i[y(0) - Y]\} e^{-i\omega_c t}. \quad (5)$$

Using the fact that the complex function $[x - X] + i[y - Y] = B(t)e^{iG(t)}$, with $B(t)$ and $G(t)$ real, Eq. (5) becomes

$$[x(t) - X] + i[y(t) - Y] = B(t)e^{iG(t)} = B(0)e^{i[G(0) - \omega_c t]}. \quad (6)$$

Thus, $x(t)$ and $y(t)$ define a point on a circle in the complex plane, centered about $X + iY$. The circle has radius $B(0) = |[x(0) - X] + i[y(0) - Y]| = \sqrt{[x(0) - X]^2 + [y(0) - Y]^2}$. The point (x, y) evolves from point $(x(0), y(0))$, which corresponds to the complex number $B(0)e^{iG(0)}$, to $(x(t), y(t))$, which corresponds to the number $B(t)e^{iG(t)}$. The trajectory followed is clockwise on the circle. This is clear from the last step in Eq. (6): the original complex number $B(0)e^{iG(0)} \rightarrow B(0)e^{i[G(0) - \omega_c t]}$. The magnitude of $B(t)e^{iG(t)}$ is always $B(0)$, but the polar angle $G(0) - \omega_c t$ continually gets "smaller" – i.e., the point $(x(t), y(t))$ traverses the circle in the clockwise direction.

This circular motion in the $x - y$ plane is superposed with constant velocity v_z in the z direction. The resulting trajectory is a helix.