Homework 2

CS 541
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A partial order $(D, \sqsubseteq)$ is *flat* if $\forall x, y \in D \left[x \sqsubseteq y \implies x = \bot_D \vee x = y \right]$. Prove the following lemma and theorems.

1 **Lemma**. Let $(D, \sqsubseteq)$ be a CPO and $f\colon D \to D$ be monotone.

$$\forall n \geqslant 0 \left[f^n(\bot) \sqsubseteq f^{n+1}(\bot) \right].$$

2 **Theorem**. $(D, \sqsubseteq_D), (E, \sqsubseteq_E)$ are flat $\implies$

a) $(D \otimes E, \sqsubseteq)$ is flat
$\wedge$ b) $(D \oplus E, \sqsubseteq)$ is flat
$\wedge$ c) $(D^*, \sqsubseteq)$ is flat.

3 **Theorem**. $(D, \sqsubseteq_D), (E, \sqsubseteq_E)$ are CPOs $\implies$

a) $(D \circ\!\!\to E, \sqsubseteq)$ is a CPO
$\wedge$ b) $(D \otimes E, \sqsubseteq)$ is a CPO
$\wedge$ c) $(D \oplus E, \sqsubseteq)$ is a CPO.