

Homework 2

CS 541
Fall 1999
Craig A. Rich

A partial order (D, \sqsubseteq) is flat if $\forall x, y \in D, x \sqsubseteq y \implies x = \perp_D \vee x = y$. Prove the following lemma and theorems.

1 **Lemma.** Let (D, \sqsubseteq) be a CPO and $f: D \rightarrow D$ be monotone.

$$\forall n \geq 0, f^n(\perp) \sqsubseteq f^{n+1}(\perp).$$

2 **Theorem.** $(D, \sqsubseteq_D), (E, \sqsubseteq_E)$ are flat \implies a) $(D \otimes E, \sqsubseteq)$ is flat
 \wedge b) $(D \oplus E, \sqsubseteq)$ is flat
 \wedge c) (D^*, \sqsubseteq) is flat.

3 **Theorem.** $(D, \sqsubseteq_D), (E, \sqsubseteq_E)$ are CPOs \implies a) $(D \multimap E, \sqsubseteq)$ is a CPO
 \wedge b) $(D \otimes E, \sqsubseteq)$ is a CPO
 \wedge c) $(D \oplus E, \sqsubseteq)$ is a CPO.