

Homework 1

CS 541
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Let (D, \sqsubseteq) be a poset. Prove the following lemma and theorem.

1 **Lemma.** $M \subseteq D$ is finite and directed $\implies \exists x \in M, x = \sqcup M$.

2 **Theorem.** D is finite $\wedge \exists$ least element $\perp_D \in D \implies (D, \sqsubseteq)$ is a CPO.

Let (D, \sqsubseteq) and (E, \sqsubseteq) be CPOs. Prove the following lemmas.

3 **Lemma.** $f: D \rightarrow E$ is continuous $\implies f$ is monotone.

4 **Lemma.** $f: D \rightarrow E$ is monotone $\wedge D$ is finite $\implies f$ is continuous.