Midterm

CS 541
Fall 1991
Craig A. Rich

In problems 1–3, refer to the denotational semantics specification for constant declarations on pp. 26–27 of the notes.

1 Compute $\mathcal{E}[\texttt{2*x}](\text{binding}\,\mathcal{I}[\texttt{x}]\ 3)$.

2 Compute $\mathcal{E}[\texttt{let val x=true in } \neg\texttt{x}]$.

3 Use the fact that overlay is associative, i.e.,

$$\text{overlay}\big(e_2, \text{overlay}(e_1, e_0)\big) = \text{overlay}\big(\text{overlay}(e_2, e_1), e_0\big),$$

to show that

$$\mathcal{E}[\texttt{let } CD_1 \texttt{ in let } CD_2 \texttt{ in } E] = \mathcal{E}[\texttt{let } CD_1\texttt{; } CD_2 \texttt{ in } E].$$

In problems 4–5, let $(D_1, \sqsubseteq_{D_1})$, $(D_2, \sqsubseteq_{D_2})$, $(D_3, \sqsubseteq_{D_3})$ be CPOs, $e_1\colon D_2 \to D_3$, $e_2\colon D_1 \to D_2$. Prove the following theorems.

4 **Theorem**. $\forall$ directed $M \subseteq D_1 \times D_2,\ \exists y \in D_1 \times D_2,\ y = \sqcup M$.

5 **Theorem**. e_1, e_2 are continuous $\Longrightarrow e_1 \circ e_2$ is continuous.

6 Design a semantic function $\mathcal{N}\colon$Nested-Parentheses $\to \mathbf{N}_\perp$ for the following syntax so that the denotation of Nested-Parentheses is the maximum depth of parenthesis nesting. For example, $\mathcal{N}[\texttt{(()())()}] = 2$.

- Syntax

$$N \quad\longrightarrow\quad \varepsilon \mid (N) \mid N_1 N_2 \qquad \text{(Nested-Parentheses)}$$