

1 Prove or disprove the following conjecture:

Conjecture. If L is the natural loop of back edge (n, d) , then n dominates all exit nodes of L .

2 The algorithm for constructing the natural loop of a back edge (n, d) (notes, p. 28, part 2) uses depth-first search to find nodes m such that there is a path from m to n not containing d . Implement the algorithm using breadth-first search instead of depth-first search.

3 The algorithm for code motion from a loop L (notes, p. 32) moves a loop-invariant definition s (defining variable x) to the pre-header only if s is the only definition of x that reaches uses of x in L . Give an example of a loop containing a loop-invariant definition s that satisfies all conditions for code motion except this one.

4 Consider the following basic block in which a , b , c , and d are live at the end of the block:

$a := 2$
$b := a * 3$
$c := x[b]$
$d := a * 3$
$c := c + b$
$c := b * d$

- Apply the four optimizations—common subexpression elimination (CSE), constant folding (CF), copy propagation (CP), and dead-code elimination (DCE)—until no further applications can be made. The optimizations can be applied in any order. Show the resulting basic block after each application.
- Use the DAG construction algorithm to generate “optimized” three-address code. Show the DAG which is constructed, indicating the primary label at each node.

5 Prove the following theorem:

Theorem. Every inner loop contains a natural loop.

6 Prove the following theorem:

Theorem. There exists a natural loop L of a back edge (n, d) such that dom is not a total order on L .