Final Exam

CS 540
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Solve any five problems from among problems 1–6.

1 Apply strength reduction and induction variable elimination to the loop in the following flow graph. Show all induction variables and the resulting flow graph after each optimization.

2 Let $(D, \leq)$ be a complete lattice, $A \subseteq D$. Prove there exists exactly one $x \in D$ such that x is a least upper bound of A.

Problems 3 and 5 refer to the following flow graph $G = (V, E)$:

3 Apply live variable analysis to the flow graph G. Show the values of OUT$[B]$ and IN$[B]$ after initialization and each complete iteration of the while-loop. Process nodes in the for-loops by decreasing node number.

Problems 4 and 5 refer to the following definitions:

- VARS is the set of variables in a flow graph;
- $D = \{\, \gamma \mid \gamma\colon \text{VARS} \to \wp(\text{VARS}) \,\}$;
- $\leq$ is the binary relation on D defined by $\gamma \leq \delta \iff \forall x \in \text{VARS},\ \gamma(x) \supseteq \delta(x)$;
- For each assignment s of the form

$$x \mathrel{:=} y,\ x \mathrel{:=} \mathrm{op}\, y,\ \text{or}\ x \mathrel{:=} y\, \mathrm{op}\, z,$$

$f_s\colon D \to D$ is defined by $f_s(\gamma) = \delta$, where

$$\delta(v) = \begin{cases} \gamma(v), & \text{if } v \neq x, \\ \{y\} \cup \gamma(y),\ \{y\} \cup \gamma(y),\ \text{or}\ \{y, z\} \cup \gamma(y) \cup \gamma(z), & \text{if } v = x; \end{cases}$$

- $I\colon D \to D$ is defined by $I(\gamma) = \gamma$;
- F is the closure under composition of $\{I\} \cup \{\, f_s \mid s \text{ is an assignment statement} \,\}$.
- The *value dependency framework* is $(D, \leq, F)$.

4 Prove that the value dependency framework is a distributive data flow framework.

5 Apply value dependency analysis to the flow graph G defined above. For each node $B \in V$, the transfer function $f_B\colon D \to D$ is defined by

$$f_B = I \circ f_{s_1} \circ f_{s_1} \circ \cdots \circ f_{s_d},$$

where $s_1, s_2, \ldots, s_d$ are the assignment statements in B. Show the values of IN$[B]$ and OUT$[B]$ after initialization and each complete iteration of the while loop. Process nodes in the for-loops by increasing node number.

6 Let $G = (V, E)$ be a flow graph. Let $(D, \leq, F)$ be a monotonic data flow framework such that for each $B \in V$, there is a transfer function $f_B \in F$. Prove that $\forall k \geq 0$ $\forall B \in V$,

$$\text{IN}_{k+1}[B] \leq \text{IN}_k[B].$$

Use induction on k and assume computation of IN during iteration $k+1$ uses values of OUT computed during iteration k.