

Problems 1–3 refer to the following three-address program:

```
    v := 2
    w := 1
    x := 1
    if z > 50 goto (D)
(A) w := w + 5
    y := v + 2
    if w > 10 goto (C)
(B) z := x + 3
    if x < 20 goto (A)
(C) w := w - 7
    x := 5 * w
    y := 2 * v
    if w ≠ 0 goto (B)
(D) continue
```

- 1 Construct the flow graph which corresponds to the three-address program, and identify the loop in the flow graph.
- 2
 - a) Apply strength reduction to the flow graph loop in problem 1. Show all induction variables and the resulting flow graph.
 - b) Apply induction variable elimination to the flow graph loop in a). Show the resulting flow graph.
- 3 Apply constant value analysis to the flow graph in problem 1. Define f_B for each block B . Show the values of $IN[B]$ and $OUT[B]$ after each complete iteration of the while-loop.

4 Let L be a set of nodes in a flow graph. Prove that L is an inner loop implies L is a natural loop.

5 Let (D, \leq) be a complete lattice, $A \subseteq D$. Prove there exists exactly one $x \in D$ such that x is a greatest lower bound of A .

6 Let $G = (V, E)$ be a flow graph. Let (D, \leq, F) be a monotonic data flow framework such that for each $B \in V$, there is a transfer function $f_B \in F$. Prove that $\forall k \geq 0$
 $\forall B \in V$,

$$\text{IN}_{k+1}[B] \leq \text{IN}_k[B].$$

Use induction on k and assume computation of IN during iteration $k + 1$ uses values of OUT computed during iteration k .