

**Homework 5**CS 531  
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A language  $L \subseteq \Sigma^*$  is in the class co-NP if and only if its complement  $\Sigma^* - L$  is in NP; i.e.,

$$\text{co-NP} =_{\text{defn}} \{ L \mid \Sigma^* - L \in \text{NP} \}.$$

Prove the following theorems.

1 **Theorem.**  $P \subsetneq \text{NP}$  and  $L$  is NP-complete  $\implies L \notin P$ .

2 **Theorem.**  $L \in P \iff \Sigma^* - L \in P$ .

3 **Theorem.**  $\text{NP} \neq \text{co-NP} \implies P \neq \text{NP}$ .

4 **Theorem.**  $L_1 \leq_m^P L_2 \iff \Sigma^* - L_1 \leq_m^P \Sigma^* - L_2$ .

5 **Theorem.**  $B \in \text{co-NP}$  and  $B$  is NP-complete  $\implies \text{NP} = \text{co-NP}$ .

6 **Theorem.**  $L_1 \leq_m^P L_2$ , where

$$\begin{aligned} L_1 &= \{ x \mid x \in \{0, 1\}^* \text{ and } x \text{ contains an even number of 1s} \}, \\ L_2 &= \{ x \mid x \in \{0, 1\}^* \text{ and } x \text{ contains an odd number of 1s} \}. \end{aligned}$$

Note: Prove this theorem by designing a complete DTM that witnesses  $L_1 \leq_m^P L_2$ .