

Homework 4

CS 531
Winter 2005
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A language L_1 is *many-one equivalent* to L_2 in polynomial time, denoted $L_1 \equiv_m^P L_2$, if $L_1 \leq_m^P L_2$ and $L_2 \leq_m^P L_1$. Prove the following theorems.

- 1 **Theorem.** $L_1 \leq_m^P L_2$ and $L_2 \leq_m^P L_3 \implies L_1 \leq_m^P L_3$.
- 2 **Theorem.** L_1 is NP-hard and $L_1 \leq_m^P L_2 \implies L_2$ is NP-hard.
- 3 **Theorem.** L_1 is NP-complete and $L_1 \leq_m^P L_2$ and $L_2 \in \text{NP} \implies L_2$ is NP-complete.
- 4 **Theorem.** L_1 is NP-complete and L_2 is NP-complete $\implies L_1 \equiv_m^P L_2$.
- 5 **Theorem.** $P = \text{NP}$ and $L_1 \in \text{NP}$ and $L_2 \in \text{NP} \implies L_1 \equiv_m^P L_2$.