Homework 3

CS 531
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For each of the following decision problems,

a) Give a reasonable length function,
b) Give a pseudo-code algorithm which correctly accepts instances for which the answer to the question is yes, and
c) Give an upper bound on the worst-case time or space complexity of your algorithm—as a function of the length of an instance (defined by the length function)—to justify your result.

1 Show that **TSP** $\in$ NP.

Traveling Salesman (TSP)

Instance: A finite set $C = \{c_1, c_2, \ldots, c_m\}$ of "cities", a "distance" $d(c_i, c_j) \in \mathbf{Z}^+$ for each pair of cities $c_i, c_j \in C$, and a bound $B \in \mathbf{Z}^+$ (where $\mathbf{Z}^+$ denotes the positive integers).

Question: Is there a "tour" of all the cities in C having total length no more than B, i.e., an ordering $\langle c_{\pi(1)}, c_{\pi(2)}, \ldots, c_{\pi(m)} \rangle$ of C such that

$$\left(\sum_{i=1}^{m-1} d(c_{\pi(i)}, c_{\pi(i+1)})\right) + d(c_{\pi(m)}, c_{\pi(1)}) \leqslant B?$$

2 Show that **SAT** $\in$ NP.

Satisfiability (SAT)

Instance: A set U of variables and a collection C of clauses over U.

Question: Is there a satisfying truth assignment for C?

3 Show that **QBF** $\in$ PSPACE.

Quantified Boolean Formulas (QBF)

Instance: A well-formed quantified Boolean formula $F = (Q_1 x_1)(Q_2 x_2) \cdots (Q_n x_n) E$, where E is a Boolean expression involving the variables $x_1, x_2, \ldots, x_n$ and each Q_i is either "$\exists$" or "$\forall$".

Question: Is F true?

4 Show that **GAP** $\in$ NL.

Graph Accessibility Problem (GAP)

Instance: An undirected graph $G = (V, E)$, and vertices $a, b \in V$.

Question: Is there a path in G from a to b?