Homework 2

CS 531
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In proving these theorems, reference all theorems (12.1–12.11) that are applied. Equality ($=$) is proved by showing inclusion ($\subseteq$) and containment ($\supseteq$). Proper inclusion ($\subsetneq$) is proved by showing inclusion ($\subseteq$) and non-containment ($\not\supseteq$).

1 Prove the following theorems.

a) **Theorem**. $\bigcup_{c \geqslant 1} \mathit{DSPACE}(\log^c n) = \bigcup_{c \geqslant 1} \mathit{NSPACE}(\log^c n)$.
This complexity class is called POLYLOGSPACE.

b) **Theorem**. $\bigcup_{c \geqslant 1} \mathit{DSPACE}(n^c) = \bigcup_{c \geqslant 1} \mathit{NSPACE}(n^c)$.
This complexity class is called PSPACE.

2 Prove the following theorems.

a) **Theorem**.
$$\begin{aligned} \mathit{DSPACE}(\log n) &\subseteq \bigcup_{c \geqslant 1} \mathit{DTIME}(n^c) \\ &\subseteq \bigcup_{c \geqslant 1} \mathit{NTIME}(n^c) \\ &\subseteq \text{PSPACE}. \end{aligned}$$
These complexity classes are called DL $\subseteq$ P $\subseteq$ NP $\subseteq$ PSPACE.

b) **Theorem**. DL $\subsetneq$ PSPACE.

3 Prove the following theorems for $1 \leqslant c < d$.

a) **Theorem**. $\mathit{DSPACE}(\log^c n) \subsetneq \mathit{DSPACE}(\log^d n)$.
b) **Theorem**. $\mathit{NSPACE}(\log^c n) \subsetneq \mathit{NSPACE}(\log^d n)$.
c) **Theorem**. $\mathit{DSPACE}(n^c) \subsetneq \mathit{DSPACE}(n^d)$.
d) **Theorem**. $\mathit{NSPACE}(n^c) \subsetneq \mathit{NSPACE}(n^d)$.
e) **Theorem**. $\mathit{DTIME}(n^c) \subsetneq \mathit{DTIME}(n^d)$.

4 Express the strongest relationship, if any, between each of the following pairs of complexity classes.

a) $\mathit{DSPACE}(n^2)$ and $\mathit{DSPACE}(f(n))$, where $f(n) = \begin{cases} n, & \text{if } n \text{ is odd;} \\ n^3, & \text{if } n \text{ is even.} \end{cases}$

b) $\mathit{DTIME}(2^n)$ and $\mathit{DTIME}(3^n)$.

c) $\mathit{NSPACE}(2^n)$ and $\mathit{DSPACE}(5^n)$.