Homework 1

CS 531
Winter 2005
Craig A. Rich

1 Show that the encoding schemes e_1, e_2, and e_3 defined on page 7 of the notes are all polynomially related.

2 Define a length function for instances G that are undirected graphs and show it is "reasonable" by showing it is polynomially related to encoding scheme e_1, e_2, or e_3 on page 7 of the notes.

3 For each of the following decision problems (from pages 46–47 of Garey and Johnson), define a simple "reasonable" length function. It is not necessary to show it is polynomially related to any particular encoding scheme.

a) **3-Satisfiability (3SAT)**
Instance: Collection $C = \{c_1, c_2, \ldots, c_m\}$ of clauses on a finite set U of variables such that $|c_i| = 3$ for $1 \leqslant i \leqslant m$.
Question: Is there a truth assignment for U that satisfies all the clauses in C?

b) **3-Dimensional Matching (3DM)**
Instance: A set $M \subseteq W \times X \times Y$, where W, X and Y are disjoint sets having the same number q of elements.
Question: Does M contain a *matching*, that is, a subset $M' \subseteq M$ such that $\|M'\| = q$ and no two elements of M' agree in any coordinate?

c) **Vertex Cover (VC)**
Instance: A graph $G = (V, E)$ and a positive integer $K \leqslant \|V\|$.
Question: Is there a *vertex cover* of size K or less for G, that is, a subset $V' \subseteq V$ such that $\|V'\| \leqslant K$ and, for each edge $\{u, v\} \in E$, at least one of u and v belongs to V'?

d) **Clique**
Instance: A graph $G = (V, E)$ and a positive integer $J \leqslant \|V\|$.
Question: Does G contain a *clique* of size J or more, that is, a subset $V' \subseteq V$ such that $\|V'\| \geqslant J$ and every two vertices in V' are joined by an edge in E?

e) **Hamiltonian Circuit (HC)**
Instance: A graph $G = (V, E)$.
Question: Does G contain a Hamiltonian circuit, that is, an ordering $\langle v_1, v_2, \ldots, v_n \rangle$ of the vertices of G, where $n = \|V\|$, such that $\{v_n, v_1\} \in E$ and $\{v_i, v_{i+1}\} \in E$ for all i, $1 \leqslant i \leqslant n$?

f) **Partition**
Instance: A finite set A and a *size* $s(a) \in \mathbf{Z}^+$ for each $a \in A$.
Question: Is there a subset $A' \subseteq A$ such that $\sum_{a \in A'} s(a) = \sum_{a \in A - A'} s(a)$?