Final Exam

CS 531
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In problems 1–5, prove the given theorem.

1. Prove that POLYLOGSPACE is closed under complement; i.e.,

   Theorem. $L \in \text{POLYLOGSPACE} \iff \Sigma^* - L \in \text{POLYLOGSPACE}$.

2. Let $L_1 \subseteq \Sigma^*$, $L_2 \subseteq \Gamma^*$.

   Theorem. $L_1 \in P$ and $L_2 \neq \phi$ and $L_2 \neq \Gamma^* \implies L_1 \leqslant_m^P L_2$.

3. A binary relation $\leqslant$ is *reflexive* if $\forall L_1, L_2\, [L_1 \leqslant L_2 \iff L_2 \leqslant L_1]$.

   A binary relation $\leqslant$ is *anti-symmetric* if $\forall L_1, L_2\, [L_1 \leqslant L_2$ and $L_2 \leqslant L_1 \implies L_1 = L_2]$.

   Theorem. $\leqslant_m^P$ is not reflexive and is not anti-symmetric.

4. **Theorem**. L_1 is NP-complete and $L_1 \equiv_m^P L_2 \implies L_2$ is NP-complete.

5. **Theorem**. L is $\mathcal{C}_2$-complete and $\mathcal{C}_1 \subseteq \mathcal{C}_2 \implies L$ is $\mathcal{C}_1$-hard.

6. In the proof that SAT is NP-complete, a language $L \in NP$ is reduced to SAT using a reduction function $f_L \colon \Sigma^* \to D_{SAT}$ defined on page 48 of the notes. Since no DTM is explicitly given to prove that f_L is computed by a polynomially time-bounded DTM, by what argument technique do we conclude that one exists?

7. $\equiv_m^P$ is an equivalence relation on $\wp(\Sigma^*)$, so it partitions $\wp(\Sigma^*)$ into classes containing "equally complex" problems, e.g., the problems in P or the NP-complete problems. How could the definition of $\leqslant_m^P$ be modified to produce a relation $\leqslant$ wherein $\equiv$ ($\leqslant$ and $\geqslant$) refines the partition induced by $\equiv_m^P$, e.g., to distinguish problems in P that are not "equally complex"?