Midterm

CS 531
Winter 2000
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1. Two functions $f_1, f_2 : \mathbf{Z}^+ \to \mathbf{R}$ are *polynomially related* if there exist polynomials p_1, p_2 such that for every $n \in \mathbf{Z}^+$, $f_1(n) \le p_2(f_2(n))$ and $f_2(n) \le p_1(f_1(n))$. Which pairs of functions from the following list are polynomially related? You need not give any proof to justify your answers.

$$n,\ \left(\tfrac{3}{2}\right)^n,\ \lg n,\ \sqrt{n},\ \lg^2 n,\ 2^n,\ n/\lg n,\ \lg\lg n,\ n^2.$$

2. Define a simple "reasonable" length function for an instance that is an NTM $M = (Q, \Sigma, \Gamma, \delta, q_0, q_Y, q_N, B)$, as defined in the notes.

3. Prove that $\mathit{DTIME}(n^2)$ is closed under union; i.e.,

$$L_1 \in \mathit{DTIME}(n^2) \text{ and } L_2 \in \mathit{DTIME}(n^2) \Longrightarrow L_1 \cup L_2 \in \mathit{DTIME}(n^2).$$

4. Prove that if $c > 1$ and $\epsilon > 0$, then $\mathit{DSPACE}(c^n) \subset \mathit{DSPACE}((c+\epsilon)^n)$.

5. Prove that if $c \ge 1$, then $\mathit{DSPACE}(2^n) \not\subseteq \mathit{NSPACE}(n^c)$.

6. Use Theorem 12.7 to prove there exists an infinite sequence $T_1(n)$, $T_2(n)$, $T_3(n)$, ... of time bounds such that

$$\mathit{DTIME}(T_1(n)) \subset \mathit{DTIME}(T_2(n)) \subset \mathit{DTIME}(T_3(n)) \subset \cdots$$

Hint: $L \in \mathit{REC}$ implies there is a time bound $T(n)$ such that $L \in \mathit{DTIME}(T(n))$.