Midterm

CS 531
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1 Define a simple "reasonable" length function for the following decision problem. It is not necessary to show that it is polynomially related to any particular encoding scheme.

Minimum Spanning Tree

Instance: An undirected graph $G = (V, E)$;
a weight function $w\colon E \to \mathbf{Z}^+$;
$B \in \mathbf{Z}^+$ such that $0 < B \leq \sum_{e \in E} w(e)$.

Question: Is there a subset $E' \subseteq E$ such that the subgraph $G' = (V, E')$ is acyclic and connected and $\sum_{e \in E'} w(e) \leq B$?

2 Let Π be a decision problem and e_1, e_2 be polynomially related encoding schemes. Prove that $L(\Pi, e_1) \in \mathrm{P} \iff L(\Pi, e_2) \in \mathrm{P}$.

3 Prove that for $k \geq 1$, $\mathit{NSPACE}\,(\lg^k n) \subseteq \bigcup_{c \geq 1} \mathit{DTIME}\,(n^{c \lg^{k-1} n})$.

4 Prove that NL $\subseteq$ P. Hint: Problem 3 is a generalization of this problem.

5 Prove there exists an infinite sequence $S_1(n)$, $S_2(n)$, $S_3(n)$, ... of space bounds such that for $i \geq 1$,

$$\mathit{DSPACE}\,(S_i(n)) \subset \mathit{DSPACE}\,(S_{i+1}(n)) \text{ and}$$
$$\mathit{NSPACE}\,(S_i(n)) \subset \mathit{NSPACE}\,(S_{i+1}(n)) \text{ and}$$
$$\mathit{DSPACE}\,(S_i(n)) \subsetneq \mathit{NSPACE}\,(S_{i+1}(n)) \text{ and}$$
$$\mathit{NSPACE}\,(S_i(n)) \subsetneq \mathit{DSPACE}\,(S_{i+1}(n)).$$

6 Consider the following decision problem:

Longest Common Superstring

Instance: A finite alphabet Σ;
A finite set $R \subseteq \Sigma^*$;
$K \in \mathbf{Z}^+$ such that $K > 0$.

Question: Is there a string $w \in \Sigma^*$ such that $|w| \leq K$ and each $x \in R$ is a substring of w?

Prove that *Longest Common Superstring* $\in$ NP.