

Homework 4

CS 531
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A language L_1 is *many-one equivalent* to L_2 in polynomial time, denoted $L_1 \equiv_m^P L_2$, if $L_1 \leq_m^P L_2$ and $L_2 \leq_m^P L_1$. Prove the following theorems.

1 **Theorem.** $L_1 \leq_m^P L_2 \wedge L_2 \leq_m^P L_3 \implies L_1 \leq_m^P L_3$.

2 **Theorem.** L_1 is NP-hard $\wedge L_1 \leq_m^P L_2 \implies L_2$ is NP-hard.

3 **Theorem.** L_1 is NP-complete $\wedge L_1 \leq_m^P L_2 \wedge L_2 \in \text{NP} \implies L_2$ is NP-complete.

4 **Theorem.** L_1, L_2 are NP-complete $\implies L_1 \equiv_m^P L_2$.

5 **Theorem.** $\text{P} = \text{NP} \wedge L_1, L_2 \in \text{NP} \implies L_1 \equiv_m^P L_2$.