

Homework 2

CS 531
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In proving these theorems, reference all theorems (12.1–12.11) which are applied. Equality ($=$) is proved by showing inclusion (\subseteq) and containment (\supseteq). Proper inclusion (\subset) is proved by showing inclusion (\subseteq) and non-containment ($\not\supseteq$).

1 Prove the following theorems.

a) **Theorem.**

$$\bigcup_{c \geq 1} DSPACE(\log^c n) = \bigcup_{c \geq 1} NSPACE(\log^c n).$$

This complexity class is called POLYLOGSPACE.

b) **Theorem.**

$$\bigcup_{c \geq 1} DSPACE(n^c) = \bigcup_{c \geq 1} NSPACE(n^c).$$

This complexity class is called PSPACE.

2 Prove the following theorems.

a) **Theorem.**

$$\begin{aligned} DSPACE(\log n) &\subseteq \bigcup_{c \geq 1} DTIME(n^c) \\ &\subseteq \bigcup_{c \geq 1} NTIME(n^c) \\ &\subseteq PSPACE. \end{aligned}$$

These complexity classes are called $DL \subseteq P \subseteq NP \subseteq PSPACE$.

b) **Theorem.** $DL \subset PSPACE$.

3 Prove the following theorems for $1 \leq c < d$.

- a) **Theorem.** $DSPACE(\log^c n) \subset DSPACE(\log^d n)$.
- b) **Theorem.** $NSPACE(\log^c n) \subset NSPACE(\log^d n)$.
- c) **Theorem.** $DSPACE(n^c) \subset DSPACE(n^d)$.
- d) **Theorem.** $NSPACE(n^c) \subset NSPACE(n^d)$.
- e) **Theorem.** $DTIME(n^c) \subset DTIME(n^d)$.

4 Express the strongest relationship, if any, between each of the following pairs of complexity classes.

a) $DSPACE(n^2)$ and $DSPACE(f(n))$, where

$$f(n) = \begin{cases} n, & \text{if } n \text{ is odd;} \\ n^3, & \text{if } n \text{ is even.} \end{cases}$$

b) $DTIME(2^n)$ and $DTIME(3^n)$.

c) $NSPACE(2^n)$ and $DSPACE(5^n)$.