Final Exam

CS 531
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Craig A. Rich

Prove the following theorems. The symbol $\subset$ denotes proper subset.

1 **Theorem**. *GAP* $\leq_{\mathrm{m}}^{\mathrm{P}}$ *Traveling Salesman*, where *GAP* and *Traveling Salesman* are defined by

Graph Accessibility Problem (GAP)

Instance: An undirected graph $G = (V, E)$;
$a, b \in V$.

Question: Is there a path in G from a to b?

Traveling Salesman

Instance: A finite set of cities $C = \{c_1, c_2, \ldots, c_m\}$;
A distance function $d\colon C \times C \to \mathbf{Z}^+$;
A bound $B \in \mathbf{Z}^+$.

Question: Is there a permutation $\pi\colon \{1, \ldots, m\} \to \{1, \ldots, m\}$ such that

$$\sum_{i=1}^{m-1} d(c_{\pi(i)}, c_{\pi(i+1)}) + d(c_{\pi(m)}, c_{\pi(1)}) \leq B?$$

2 **Theorem**. *Vertex Cover* $\leq_{\mathrm{m}}^{\mathrm{P}}$ *Dominating Set*, where *Vertex Cover* and *Dominating Set* are defined by

Vertex Cover

Instance: An undirected graph $G = (V, E)$;
$K \in \mathbf{Z}^+$ such that $K \leq \|V\|$.

Question: Is there a $V' \subseteq V$ such that $\|V'\| \leq K$
and for every $\{u, v\} \in E$, $u \in V'$ or $v \in V'$?

Dominating Set

Instance: An undirected graph $G = (V, E)$;
$K \in \mathbf{Z}^+$ such that $K \leq \|V\|$.

Question: Is there a $V' \subseteq V$ such that $\|V'\| \leq K$
and for every $u \in V - V'$ there is a $v \in V'$ such that $\{u, v\} \in E$?

3 **Theorem**. $L(\Pi, e_1) \equiv^{\mathrm{P}}_{\mathrm{m}} L(\Pi, e_2)$, where Π is a decision problem and e_1, e_2 are polynomially related encoding schemes.

4 **Theorem**. L is E-complete $\Longrightarrow L \notin$ P.

5 **Theorem**. There exist time bounds $T_1(n)$ and $T_2(n)$ such that

(1) $\mathit{DTIME}\,(T_1(n)) \subset \mathit{DTIME}\,(T_2(n))$ and
(2) for every $L_1 \in \mathit{DTIME}\,(T_1(n))$ and $L_2 \in \mathit{DTIME}\,(T_2(n))$, $L_1 \equiv^{\mathrm{P}}_{\mathrm{m}} L_2$.

6 **Theorem**. NP $\neq$ co-NP $\Longrightarrow$ SAT $\notin$ P.