**Homework 2**

CS 531
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In proving these theorems, reference all theorems (12.1–12.11) which are applied. Equality ($=$) is proved by showing inclusion ($\subseteq$) and containment ($\supseteq$). Proper inclusion ($\subset$) is proved by showing inclusion ($\subseteq$) and non-containment ($\not\supseteq$).

**1** Prove the following theorems.

a) **Theorem**.

$$\bigcup_{c \geq 1} \mathit{DSPACE}\,(\log^c n) = \bigcup_{c \geq 1} \mathit{NSPACE}\,(\log^c n).$$

This complexity class is called POLYLOGSPACE.

b) **Theorem**.

$$\bigcup_{c \geq 1} \mathit{DSPACE}\,(n^c) = \bigcup_{c \geq 1} \mathit{NSPACE}\,(n^c).$$

This complexity class is called PSPACE.

**2** Prove the following theorems.

a) **Theorem**.

$$\begin{aligned} \mathit{DSPACE}\,(\log n) &\subseteq \bigcup_{c \geq 1} \mathit{DTIME}\,(n^c) \\ &\subseteq \bigcup_{c \geq 1} \mathit{NTIME}\,(n^c) \\ &\subseteq \text{PSPACE}. \end{aligned}$$

These complexity classes are called DL $\subseteq$ P $\subseteq$ NP $\subseteq$ PSPACE.

b) **Theorem**. DL $\subset$ PSPACE.

**3** Prove the following theorems for $1 \leq c < d$.

a) **Theorem**. $\mathit{DSPACE}\,(\log^c n) \subset \mathit{DSPACE}\,(\log^d n)$.
b) **Theorem**. $\mathit{NSPACE}\,(\log^c n) \subset \mathit{NSPACE}\,(\log^d n)$.
c) **Theorem**. $\mathit{DSPACE}\,(n^c) \subset \mathit{DSPACE}\,(n^d)$.
d) **Theorem**. $\mathit{NSPACE}\,(n^c) \subset \mathit{NSPACE}\,(n^d)$.
e) **Theorem**. $\mathit{DTIME}\,(n^c) \subset \mathit{DTIME}\,(n^d)$.

4 Express the strongest relationship, if any, between each of the following pairs of complexity classes.

a) $\mathit{DSPACE}\,(n^2)$ and $\mathit{DSPACE}\,(f(n))$, where

$$f(n) = \begin{cases} n, & \text{if } n \text{ is odd;} \\ n^3, & \text{if } n \text{ is even.} \end{cases}$$

b) $\mathit{DTIME}\,(2^n)$ and $\mathit{DTIME}\,(3^n)$.

c) $\mathit{NSPACE}\,(2^n)$ and $\mathit{DSPACE}\,(5^n)$.