Final

CS 531
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Prove the following theorems. The symbol $\subset$ denotes proper subset.

1 **Theorem**. NL $\subset$ POLYLOGSPACE.

2 **Theorem**. $GAP \in \mathrm{P}$, where GAP is defined by

Graph Accessibility Problem (GAP)

Instance: An undirected graph $G = (V, E)$;
$a, b \in V$.

Question: Is there a path in G from a to b?

3 **Theorem**. *Vertex Cover* $\leq_{\mathrm{m}}^{\mathrm{P}}$ *Dominating Set*, where *Vertex Cover* and *Dominating Set* are defined by

Vertex Cover

Instance: An undirected graph $G = (V, E)$;
$K \in \mathbf{Z}^+$ such that $K \leq \|V\|$.

Question: Is there a $V' \subseteq V$ such that $\|V'\| \leq K$
and for every $\{u, v\} \in E$, $u \in V'$ or $v \in V'$?

Dominating Set

Instance: An undirected graph $G = (V, E)$;
$K \in \mathbf{Z}^+$ such that $K \leq \|V\|$.

Question: Is there a $V' \subseteq V$ such that $\|V'\| \leq K$
and for every $u \in V - V'$ there is a $v \in V'$ such that $\{u, v\} \in E$?

4 **Theorem**. B is $\mathbf{C}$-complete $\wedge\, B \in \mathrm{NP} \Longrightarrow \mathbf{C} \subseteq \mathrm{NP}$, where $\mathbf{C}$ is a class of languages.

5 **Theorem**. $L_1 \leq_{\mathrm{m}}^{\mathrm{P}} L_2 \wedge L_2 \in \text{co-NP} \Longrightarrow L_1 \in \text{co-NP}$.

6 **Theorem**. $\mathrm{NP} \neq \text{co-NP} \wedge B$ is NP-complete $\Longrightarrow B \notin \text{co-NP}$.