Homework 2

CS 450
Spring 2004
Craig A. Rich

Let Σ be a finite alphabet. A language L over Σ is a subset of Σ^*; i.e., $L \subseteq \Sigma^*$. The powerset $\wp(\Sigma^*)$ of Σ^* is the set of all subsets of Σ^* (languages over Σ); i.e.,

$$\wp(\Sigma^*) = \{ L \mid L \subseteq \Sigma^* \}.$$

$\mathcal{RE}$ and $\mathcal{REC}$ are the set of all recursively enumerable and recursive languages over Σ; i.e.,

$$\mathcal{RE} = \{ L \mid L \subseteq \Sigma^* \text{ and } L \text{ is recursively enumerable} \},$$
$$\mathcal{REC} = \{ L \mid L \subseteq \Sigma^* \text{ and } L \text{ is recursive} \}.$$

Prove the following theorems.

1. **Theorem**. $\wp(\Sigma^*)$ is uncountable.

2. **Theorem**. $\mathcal{RE}$ and $\mathcal{REC}$ are countable.

3. **Theorem**. $L \in \mathcal{RE} \implies$ L is accepted by a DTM in which every accepting configuration is a halting configuration.

4. **Theorem**. $L_1 \in \mathcal{REC}$ and $L_2 \in \mathcal{REC} \implies L_1 \cap L_2 \in \mathcal{REC}$.

5. **Theorem**. $L_1 \in \mathcal{RE}$ and $L_2 \in \mathcal{RE} \implies L_1 \cap L_2 \in \mathcal{RE}$.