**Final Exam**

CS 310
Winter 2004
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1 Give a regular expression $e$ such that $L_e = \{\, x \in \{\mathtt{a}, \mathtt{b}\}^* \mid x \text{ does not contain } \mathtt{ab} \,\}$.

2 Consider the following lexicon $\mathcal{E}$:

$$\begin{aligned} a &\rightarrow \mathtt{1(01)*} \\ b &\rightarrow \mathtt{(101)*} \\ c &\rightarrow \mathtt{(0+)1} \end{aligned}$$

a) Give the lexical NFA $\mathcal{N}(\mathcal{E})$ constructed from $\mathcal{E}$.

b) Give the lexical DFA $\mathcal{D}(\mathcal{N}(\mathcal{E}))$ constructed from $\mathcal{E}$.

c) Show the sequence of tokens and matching strings obtained by greedy grab lexical analysis on the source string $x = \mathtt{1101101011}$.

3 Let $e$ be a regular expression and $\mathcal{D}(\mathcal{N}(e)) = (Q', \Sigma, \delta', i', F')$ be the DFA constructed from the NFA constructed from $e$. Prove that

$$\widetilde{\delta'}(i', x) = \phi \implies \forall y{\in}\Sigma^* \, [xy \notin L_e].$$

Hint: use induction on $|y|$.

4 Use the pumping lemma (Martin, Theorem 5.2a) to show that

$$L = \{\, \mathtt{a}^i \mathtt{b}^j \mathtt{c}^k \mid j = i \text{ or } j = k \}$$

is not a regular language.

5 Consider the following CFG G:

$$\begin{aligned} S &\rightarrow \mathtt{a}S \mid \mathtt{a}BC \mid \mathtt{a}C \\ B &\rightarrow BB \mid \mathtt{b} \mid \varepsilon \\ C &\rightarrow \mathtt{cc} \end{aligned}$$

a) Show that G is ambiguous.

b) Give a regular expression $e$ such that $L_e = L_G$.

c) Give a CFG $G'$ such that $G'$ is unambiguous and $L_{G'} = L_G$.

6 Prove that $\mathbf{CFL}_{\{0,1\}} \not\subseteq \mathbf{REG}_{\{0,1\}}$.