Homework 2

CS 240
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1 In each of the following, give the simplest function $f(n)$ such that the function $T(n)$ expressed by the recurrence equations is big-theta of $f(n)$.

$$\text{a) } T(n) = \begin{cases} 1, & \text{if } n = 1; \\ 3T(n/2) + n, & \text{if } n > 1. \end{cases}$$

$$\text{b) } T(n) = \begin{cases} 1, & \text{if } n = 1; \\ 3T(n/2) + n^2, & \text{if } n > 1. \end{cases}$$

$$\text{c) } T(n) = \begin{cases} 1, & \text{if } n = 1; \\ 8T(n/2) + n^3, & \text{if } n > 1. \end{cases}$$

2 In each of the following, give the simplest function $f(n)$ such that the function $T(n)$ expressed by the recurrence equations is big-theta of $f(n)$.

$$\text{a) } T(n) = \begin{cases} 1, & \text{if } n = 1; \\ 4T(n/3) + n, & \text{if } n > 1. \end{cases}$$

$$\text{b) } T(n) = \begin{cases} 1, & \text{if } n = 1; \\ 4T(n/3) + n^2, & \text{if } n > 1. \end{cases}$$

$$\text{c) } T(n) = \begin{cases} 1, & \text{if } n = 1; \\ 9T(n/3) + n^2, & \text{if } n > 1. \end{cases}$$

3 Find a closed-form expression in terms of a and c for the function $T(n)$ expressed by the following recurrence equations, and give the simplest function $f(n)$ such that $T(n)$ is $\Theta(f(n))$.

$$T(n) = \begin{cases} 1, & \text{if } n = 1; \\ aT(n-1) + c, & \text{if } n > 1. \end{cases}$$

Derive the closed form by expanding the recurrence, choosing a number of decrements to produce size 1, and replacing the geometric series. The derivation should be similar to the derivation of a closed-form expression for functions $T(n)$ expressed by the "divide-and-conquer" recurrence equations, in which we expanded the recurrence, chose a number of divisions to produce size 1, and replaced the geometric series.

4 Express the worst-case running time $T(n)$ of the following function using recurrence equations.

```
function FACTORIAL (N: POSITIVE) return POSITIVE is

begin
   if N=1 then
      return 1;
   else
      return N * FACTORIAL(N-1);
   end if;
end FACTORIAL;
```

Use problem 3 to obtain the simplest function $f(n)$ such that $T(n)$ is $\Theta(f(n))$.

5 Express the worst-case running time $T(n)$ of the following function using recurrence equations.

```
procedure PERMUTATIONS (S: in out STRING; N: INTEGER) is

begin
   if N=1 then
      PUT_LINE (S);
   else
      for C in 'A'..'E' loop
         S(N) := C;
         PERMUTATIONS (S,N-1);
      end loop;
   end if;
end PERMUTATIONS;
```

Use problem 3 to obtain the simplest function $f(n)$ such that $T(n)$ is $\Theta(f(n))$.