Answers to Odd Numbered Exercises

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Chapter 1

1. $y' = -3\sin(x)$, $y'' = -3\cos(x)$. So $y'' + y = -3\cos(x) + 3\cos(x)$ on $(-\infty, \infty)$.

3. $y' = 3e^{3x}$, and $y'' = 9e^{3x}$, so $y'' - 2y' - 3y = 9e^{3x} - 6e^{3x} - 3e^{3x} - 3 = -3$.

5. $y'' = g - y'$.

7. $P' = c(P^3 - P^2)$.

9.

$$\begin{aligned} u_1' &= u_2 \\ u_2' &= u_3 \\ u_3' &= -u_1 + 2u_3 \end{aligned}$$

11.

$$\begin{aligned} u_1' &= u_2 \\ u_2' &= u_3 \\ u_3' &= u_4 \\ u_4' &= u_1 + 3u_3 \end{aligned}$$

13. Since the solution is expressed in terms of a familiar function, we can say the solution is given analytically.

Section 2.1

1. Draw the field of $y' = 5(y^2)^{1/3}$ in the window $[-6, 6] \times [-4, 4]$. Draw solutions through $(0, 0)$ and through $(-2, -1)$. What does Theorem 2.1.2 have to say about this? Explain!

Solution:

Since the partial derivative with respect to y is not continuous in the window, the theorem does not apply, and multiple solutions through a point become a possibility. Note: the fact that the hypotheses of Theorem 2.1.2 do not hold does **not** guarantee multiple solutions through some point.

3. Use Field&Solution to draw solutions through $(0, 0)$ and $(0, -0.1)$ for the equation $y' = f(x, y) = |y|^{2/5}$. Without any calculation, what can you say about $\partial f/\partial y$ and why?

Solution:

The partial $\partial f/\partial y$ cannot be continuous throughout the window otherwise there could not be multiple solutions through a point.

5. Without using the Field&Solution package, describe the behavior of solutions to $y' = (y - 1)^4$ in general terms. Is there a constant solution? Hint: What does the derivative tell you?

Solution:

$y \equiv 1$ is a constant solution. Off this line the derivative is positive, and so the solution must increase.

Section 2.2

1. By hand, carry out one step of Euler's method and the Runge–Kutta method where $y' = 2xy - y$, $y(1) = 2$, and the step size is 0.1.

Solution:

Euler: $y_1 = 2.2$. Runge–Kutta: $y_1 = 2.23256$.

In Exercises 3–4, use the package ERGraphical.

3. Consider the equation $y' = x - 2y$. Approximate the solution on $[-2, 3]$ with initial value $y_0 = 0$ using the Euler and Runge–Kutta methods with step sizes $h = 1$, $h = 0.5$, and $h = 0.1$. Compare the performance of the two methods.

Solution:

As expected, the Runge–Kutta method does a far better job of approximation, particularly with fewer subdivisions.

In each of the exercises 5–6 use the package ERNumerical.

5. Consider the equation $y' = x + 2y$. Approximate the solution on $[0, 2]$ with initial value $y_0 = 2$. Run the package for step sizes of $h = 0.1$, $h = 0.05$, $h = 0.02$, and $h = 0.01$. Record the values of both methods at $x = 1$. What approximate value are you confident in for each method and why?

Solution:

h	Euler	Runge−−Kutta
0.1	13.1841	15.875
0.05	14.3869	15.8754
0.02	15.24	15.8754
0.01	15.551	15.8754

The Euler method might suggest that the digits 15 are good, but it is still shakey.

The Runge–Kutta method stabilizes almost immediately at 15.8754.

Section 2.3.4

In Exercises 1–4 check to see if the equation is exact in the plane. If so, solve it and clearly indicate on a plot of the contour the extent of the solution through the initial condition.

1. $(2x+3)+(2y-2)y'=0$ and $y(0)=1$.

Solution:

The equation is exact. The solution is an implicit function which is part of the circle $x^2+3x+y^2-2y=-1$ passing through the point $(0,1)$.

3. $(x^2y+y)+(x^3/3+y)y'=0$ and $y(0)=1$.

Solution:

This is not exact.

Orthogonal trajectories. Each equation $y'=f(x,y)$ determines a family of curves in the plane. Exercises 5–8 ask you to find a second family of curves such that each curve of this family intersects the curves of the first family at right angles. Such a family is called the **family of orthogonal trajectories**.

5. Find the orthogonal trajectories of the family $y=cx^2$. Hint: find a first order differential equation of which the curves of this family are solutions. Do this by writing $y'=2cx$, and now substitute for c from the family.

Solution:

$y'=2y/x$, so we solve $y'=-x/(2y)$. Separating and integrating gives $y^2+x^2/2=k$. The orthogonal trajectories are ellipses.

7. Find the orthogonal trajectories of the family $y = ce^x$. Hint: see 5.

Solution:

$y' = ce^x = y$, so we must solve $y' = -1/y$. We get $y^2/2 = -x + k$. This is a family of parabolas.

Section 2.4

1. Draw a phase line diagram for the equation $y' = (y-1)(y+1)$. Mark the critical points and arrows and describe the stability of the critical points.

Solution:

The critical points are 1 and -1. The first is unstable and the second stable.

3. Draw a phase line diagram for the equation $y' = y(y-1)(y+1)$. Mark the critical points and arrows and describe the stability of the critical points.

Solution:

The critical points are -1, 0, and 1, and they are unstable, stable, and unstable in that order.

5. Draw a phase line diagram for the equation $y' = y^3 - y^2 - 6y$. Mark the critical points and arrows and describe the stability of the critical points.

Solution:

The critical points are -2, 0, and 3, and they are unstable, stable, and unstable in that order.

Section 3.1

1. Solve the system

$$\begin{aligned} x' &= -3x, \\ y' &= 2y, \end{aligned}$$

with $x(0) = -1$ and $y(0) = 2$. Hint: Each equation can be solved on its own. Check your solution by substituting in the equations. Does your solution confirm the conclusions of Theorem 3.1.1? Explain!

Solution:

$x(t) = -e^{-3t}$, $y(t) = 2e^{2t}$. The functions and partials are continuous, so the solution is unique as expected.

3. Solve the system

$$\begin{aligned} x' &= 2x + 3y, \\ y' &= 2y, \end{aligned}$$

with $x(0) = 1$ and $y(0) = 1$. Hint: Solve one equation and substitute in the other. Check your solution by substituting in the equations. Does your solution confirm the conclusions of Theorem 3.1.1? Explain!

Solution:

$x(t) = 3te^{2t} + e^{2t}$, $y(t) = e^{2t}$. The functions and partials are continuous, so the solution is unique as expected.

5. Solve the system

$$\begin{aligned} x' &= e^{t}x + 3y, \\ y' &= -y, \end{aligned}$$

with $x(0) = 1$ and $y(0) = 3$.

Solution:

$x(t) = \frac{10}{e}e^{(e^t)} - 9$, $y(t) = 3e^{-t}$.

7. Use the package 2×2System to draw the field of

$$\begin{aligned} x' &= x + y - 2, \\ y' &= x - y \end{aligned}$$

in the rectangle $[-6, 6] \times [-4, 4]$. Draw orbits for initial conditions $(-1, 3)$, $(-2, 2)$, and $(1, 1)$ at $t = 0$. Explain what is happening with this last initial condition.

Solution:

The orbit is the single point $(1, 1)$